HISSAN CENTRAL EXAMINATION - 2079 (2022)

Grade : XI **F.M.: 75**

Time : 3 hrs

COM. MATHEMATICS (0071 D)

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

GROUP A

Attempt ALL Questions. **[1 × 11 = 11]**

Write the correct option in your answer sheet.

1. The compound statement "$p \Rightarrow q$" is equivalent to
   a) $\sim p \wedge q$ b) $p \wedge q$ c) $\sim p \vee q$ d) $p \vee \sim q$

2. If $\ln(y + 2) + 2\ln x = 1$, then $y =$
   a) $1/x^2 - 2$ b) $1/x - 2$ c) $e/x^2 - 2$ d) $e/(2x) - 2$

3. In any triangle ABC, $\cos\dfrac{A}{2} =$

   a) $\sqrt{\dfrac{s(s-a)}{ab}}$ b) $\sqrt{\dfrac{s(s-b)}{ac}}$

   c) $\sqrt{\dfrac{s(s-a)}{bc}}$ d) $\sqrt{\dfrac{s(s-c)}{ab}}$

4. To solve an oblique triangle, we use
   a) Sine Law   b) Cosine Law   c) Tangent Law   d) All of these

5. If the points with position vectors $(2, 6)$, $(1, 2)$ and $(a, 10)$ are collinear, then the value of a is
   a) -8 b) 4 c) 3 d) 12

6. The focus of the parabola $x^2 = 16y$ is
   a) $(4, 0)$ b) $(0, 4)$ c) $(0, -4)$ d) $(4, 4)$

7. If the arithmetic mean is 78 and coefficient of variation is 12.3%, then the standard deviation is
   a) 10.59   b) 9.59   c) 8.59   d) 11

8. If $y = \dfrac{2^{1/x} - 1}{2^{1/x} + 1}$, then
   a) $\lim\limits_{x \to 0} y = -1$ b) $\lim\limits_{x \to 0} y = 1$
   c) $\lim\limits_{x \to 0+} y = -1$ d) $\lim\limits_{x \to 0+} y = \lim\limits_{x \to 0-} y$

9. Value of the integral $\int \dfrac{e^x}{\left(1 - e^x\right)^2}\, dx$ is

   a) $-\dfrac{1}{1 - e^x} + C$ b) $-\dfrac{1}{2\left(1 - e^x\right)} + C$

   c) $\dfrac{1}{1 + e^x} + C$ d) $\dfrac{1}{1 - e^x} + C$

10. A list of values for the function $f(x) = x + 10 - e^x$ are

x	0	1	2	3	4
$f(x)$	9	8.282	4.611	-7.086	-40.598

   If the data perform two steps of the bisection method for solving $f(x) = 0$, on interval $[0, 4]$, then the root is
   a) 1   b) 2   c) 3   d) 4

11. Acceleration of train when it is moving steadily from 4.0 m/s to 20 m/s in 100s is
   a) 1 m s^{-2} b) 2 m s^{-2} c) 0.16 m s^{-2} d) 3 m s^{-2}

OR

If the price of a unit of Good 1 is Rs. 20, the price of a unit of Good 2 is Rs. 5 and the consumer's income is Rs. 100, then the intercept and slope are
a) $(20, -4)$ b) $(-20, -4)$ c) $(20, 4)$ d) $(-20, 4)$

GROUP B **[5 × 8 = 40]**

12. a) Let A and B be any two subsets of U, prove that $\overline{A \cup B} = \overline{A} \cap \overline{B}$.
    b) Solve the inequality $x^2 - 2x - 3 \geq 0$. **[2+3]**

13. a) Let $f(x) = x^3 + 2$ and $g(x) = 4x - 1$. Find the value of $fog(x)$ and $gof(x)$. Is the composite function commutative?
    b) Sketch the graph of the function $y = \log_3 x$ using its characteristics. **[2+3]**

14. a) Prove that in any $\triangle ABC$, if
$(\sin A + \sin B + \sin C)(\sin A + \sin B - \sin C) = 3 \sin A \sin B$,
then $\angle C = 60°$.
   b) Find the equation of the tangent to the circle $x^2 + y^2 = 9$ which passes through $(-3, 1)$. [3+2]

15. a) Pearson's measure of skewness of a distribution is 0.5. Its median and mode are respectively 42 and 36. Find the coefficient of variation.
   b) A problem in mathematics is given to three students A, B and C whose chances of solving it are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{2}$ respectively. What is the probability that the problem will be solved? [3+2]

16. a) Evaluate $\lim_{x \to 0} \dfrac{\tan x - \sin x}{x^3}$.
   b) Define a continuous function at $x = a$. A function $f(x)$ is defined as follows:
$$f(x) = \begin{cases} 2x + 1 & \text{for } x < 1 \\ 2 & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{cases}.$$
Is the function continuous at $x = 1$? If not, can it be made continuous at $x = 1$? Explain. [2+1+2]

17. a) Evaluate $\int \sin^2 2x \, dx$.
   b) Find the area of the region enclosed by the curve $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$ using the method of integration. [2 +3]

18. a) Find a value to $\sqrt{3}$ correct to within 10^{-2} by Newton Raphson's method.
   b) Approximate $\int_1^2 f(x)dx$ given the table of values.

x	1	5/4	3/2	7/4	2
$f(x)$	10	8	7	6	5

Compute an estimate by the composite trapezoid rule. [3+2]

19. a) State and prove triangle law of forces.
   b) A stone is dropped from a balloon at a height 116.4 m above the ground and it reaches the ground in 6 sec. Find the velocity with which the balloon was rising. [3+2]

OR

The demand and supply functions of a good are given by
$P = -Qd + 125$ and $2P = 3Qs + 30$.
Determine the equilibrium price and quantity. Determine also the effect on the market equilibrium if the government decides to impose a fixed tax of Rs 5 on each good. Who pays the tax? [2+2+1]

GROUP C [8 × 3 = 24]

20. a) A ball is dropped from a height of 128 feet. If it rebounds ¾ of the height from which if it falls every time it hits the ground, how high will it bounce after it strikes the ground for the tenth time. Also, find the total distance it travels.
   b) Find the square root of the complex number $8 - 6i$.
   c) Using properties of a determinant, prove that
$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + ab + bc + ca.$$ [3+2+3]

21. a) Prove that if P_1 and P_2 be the lengths of perpendiculars from the origin upon the straight line whose equations are $x \sec \theta + y \csc \theta = a$ and $x\cos\theta - y \sin \theta = a\cos 2\theta$, then $4P_1^2 + P_2^2 = a^2$.
   b) Find the value of k so that $2x^2 + 7xy + 3y^2 - 4x - 7y + k = 0$ may represent a pair of lines.
   c) Prove that $\cos(A + B) = \cos A \cos B - \sin A \sin B$ by vector method. [3+2+3]

22. a) Let $f(x) = \cos 4x$. Find $\dfrac{d}{dx} f(x)$ from first principle.
   b) Water flows into an inverted conical vessel at the rate of $24 \text{m}^3/\text{m}$. When the depth of water is 4m, how fast is the level rising, assuming that the height of the vessel is 8m and the radius at the top is 2m? [4+4]

THE END