HISSAN CENTRAL EXAMINATION - 2079 (2022)

Grade: XII **F.M.: 75**

Time : 3 hrs

COM. MATHEMATICS (0081 A)

Candidates are required to give their answers in their own words as far as practicable.

*Attempt **ALL** Questions.*

GROUP A **[11 × 1 = 11]**

Rewrite the correct option in your answer sheet.

1. The number $(-1)^{1/3}$ is not equal to

   a) –1 b) $(-\sqrt{3}+i)/(2i)$ c) $(\sqrt{3}+i)/(2i)$ d) $(\sqrt{3}-i)/(2i)$

2. If the one root of the equation $4x^2 - 2x + p - 4 = 0$ is the reciprocal of other, then the value of p is

   a) 8 b) – 8 c) – 4 d) 4

3. The general value of q satisfying the equation $2\sin^2 q - 3\sin q - 2 = 0$, is

   a) $n\pi + (-1)^n\,\pi/6$ b) $n\pi + (-1)^n\,\pi/3$

   c) $n\pi + (-1)^n\,5\pi/6$ d) $n\pi + (-1)^n\,7\pi/6$

4. The solution of the trigonometric equation tan x +1=2 on the interval $[0,2\pi)$ is

   a) $0, \pi$ b) $3\pi/4, 7\pi/4$ c) $\pi/4, 5\pi/4$ d) $3\pi/4, 5\pi/4$

5. If $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is

   a) π b) $\pi/2$ c) $\pi/4$ d) $\pi/3$

6. The eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a < b$ is

   a) $\sqrt{1-\frac{b^2}{a^2}}$ b) $\sqrt{\frac{b^2}{a^2}-1}$ c) $\sqrt{1-\frac{a^2}{b^2}}$ d) $\sqrt{\frac{a^2}{b^2}-1}$.

7. In a binomial distribution, if n =8 and p = 1/3, then the variance is

   a) 8/3 b) 48/3 c) 64/3 d) 16/9

8. The points on the curve $x^2 + y^2 - 2x - 3 = 0$, where the tangents are parallel to the x-axis are

   a) (–1, 2), (1, 2) b) (–1, 3), (1, 3)

   c) (1, 2), (1, – 2) d) (1, 3), (1, – 3)

9. The I.F. for differential equation $\frac{dy}{dx} + Py = Q$ is

   a) $e^{\int Q\,dx}$ b) $e^{\int P\,dx}$ c) $e^{-\int P\,dx}$ d) $e^{\int Q\,dy}$

10. You have a system of three linear equations with three unknowns. If you perform Gaussian elimination and obtain the row-reduced echelon form $\left(\begin{array}{ccc|c} 1 & -2 & 4 & 6 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$, then the system has

    a) a unique solution b) no solution

    c) infinitely many solutions d) finite number of solutions

11. Forces of magnitudes 1 N, 2 N, 3 N and 4 N are acting along sides AB, CB, DC and DA of a rectangle respectively. The magnitude of resultant is

    a) 7.21 N b) 4 N c) 6 N d) 0N

    OR

    Given the national income model $Y = E;\ E = C + I$, where $C = 280 + 0.6Y$ and $I = 80$, the values of the intercept and slope of the expenditure equation are

    a) 360, 0.6 b) 280, 0.4 c) 280, 0.6 d) 360, 0.4

GROUP B **[8 × 5 = 40]**

12. a) State De Moivre's theorem. Using it, find cube roots of – 27

    b) If the roots of $(c^2+d^2)x^2 - 2(ac+bd)x + (a^2+b^2) = 0$ are equal, prove that $bc - ad = 0$.

    [3+2]

13. a) Using mathematical induction, prove that

    $2 + 2^2 + \ldots + 2^n = 2\,(2^n - 1)$.

b) Solve the system $x + 2y + 3z = 6$, $2x + 4y + z = 7$ and $3x + 2y + 9z = 14$ by the inverse matrix method.

[2+3]

14. a) Find the value of $\cot^{-1} 3 + \operatorname{cosec}^{-1} \sqrt{5}$.

b) Determine the equation of the hyperbola in the standard position with focus at (–7, 0) and eccentricity 7/4.

[3+2]

15. a) For the observations of the variables X and Y, the following results are obtained:

$$\Sigma X = 36, \quad \Sigma Y = 48, \quad \Sigma XY = 225, \quad \Sigma Y^2 = 340, \quad n = 5.$$

Find the equation of the regression line of X on Y.

b) A dice is thrown 3 times. Getting a 2 or 3 is regarded as a success. Find the probabilities of getting two successes.

[3+2]

16. Compute the integrals

a) $\displaystyle\int \frac{dx}{a + b\cos x} \quad (|a| < |b| < 0)$

b) $\displaystyle\int \frac{dx}{(x + 1)(x + 2)(x + 3)}$.

[3+2]

17. State Bernoulli's equation. Solve the differential equation

$$\frac{dy}{dx} + \sin 2y \frac{1}{x} = x^3 \cos^2 y.$$

[1+4]

18. An experiment involves placing the males and females of a laboratory animal species in two separate controlled environments. There is a limited time available in these environments, and the experiment or wishes to maximizes the number of animals subject to the constraints described

	Males	Females	Time available
Environment A	20 min	25 min	800 min
Environment B	20 min	15 min	600 min

a) Formulate the given problem mathematically.

b) How many males and females will maximize the total number of animals? Solve the problem by Simplex Method.

[1+4]

19. a) Two men have to remove a stone of weight 180 kg wt. with a light plank of length 1.5 m. If the stronger of them is able to carry 120 kg-wt, how the stone must be placed so as to allow him that share of the weight.

b) A shot is seen to pass horizontally just over a vertical wall 19.6 m high and 39.2 m off. Find the direction and magnitude of the velocity with which the shot left the gun. ($g = 9.8$ m/s^2)

[2+3]

OR

a) Given the demand function $p_d = 300 - x$ and the supply function $p_s = x + 100$, where x is the number of units, find the consumer's as well as producer's surplus.

b) Solve the difference equation $y_t = 0.2y_{t-1} + 0.8t + 5$. Is the general solution stable or unstable?

[3+2]

GROUP C [3 × 8 = 24]

20. a) A committee of 15 is to be committed, choosing at least 5 from group A and at least 7 from group B. If there are 10 persons in each of these groups, in how many ways can the forms be constituted?

b) If $y = \frac{x}{1!} - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \ldots$, show that

$$x = y + \frac{y^2}{2} + \frac{y^3}{3} + \frac{y^4}{4} + \ldots$$

c) Given the algebraic structure $(G,*)$ with $G = \{1, \omega, \omega^2\}$ where ω represents an imaginary cube root of unity and $*$ stands for the binary operation of multiplication, show that $(G,*)$ is a group.

[3+2+3]

21. a) Prove that a line which makes angle x, y, z, t with four diagonals of a cube is $\cos^2 x + \cos^2 y + \cos^2 z + \cos^2 t = \dfrac{4}{3}$.

b) Define the vector product of two vectors and interpret it geometrically. Find the area of the parallelogram determined by the vectors $\vec{i} + 2\vec{j} + 3\vec{k}$ and $-3\vec{i} - 2\vec{j} + \vec{k}$.

[5+3]

22. a) Let $f(x) = \ln \tan x$. Find $\dfrac{d}{dx} f(x)$ from first principle.

b) Find the derivative of $x^{\cosh x}$.

c) State L'Hospital's Rule. Use it to find the value of

$$\lim_{x \to 0} \frac{e^x - e^{-x} - 2\cos x}{\sin^2 x}.$$

[4+2+2]

THE END