HISSAN CENTRAL EXAMINATION - 2079 (2022)

Grade : XII **F.M.: 75**

Time : 3 hrs

COM. MATHEMATICS (0081 B)

Candidates are required to give their answers in their own words as far as practicable.

*Attempt **ALL** Questions.*

GROUP A

Rewrite the correct option in your answer sheet. **[11× 1 = 11]**

1. The value of the expression $(-1/2 + i\sqrt{3}/2)^{637} + (-1/2 - i\sqrt{3}/2)^{637}$ is

   a) –1 b) 0 c) 1 d) i

2. If the one root of the equation $4x^2 - 2x + p - 4 = 0$ is the reciprocal of other, then the value of p is

   a) 8 b) – 8 c) – 4 d) 4

3. All solutions of the equation $\sin 2x = -\sin(-x)$ in the interval $[0, 2\pi)$ are

   a) 0 b) $0, \pi/3, \pi, 5\pi/3$ c) $0, \pi$ d) $\pi/3, \pi$

4. The general solution of the trigonometric equation $3\sec^2 x - 4 = 0$ is

   a) $\pi/3 + 2n\pi, 5\pi/3 + 2n\pi$ b) $\pi/6 + 2n\pi, 11\pi/6 + 2n\pi$

   c) $\pi/3 + n\pi, 5\pi/3 + n\pi$ d) $\pi/6 + n\pi, 11\pi/6 + n\pi$

5. If $\vec{a}$ is a unit vector and $(\vec{x} + 2\vec{a})\cdot(\vec{x} - 2\vec{a})$, then the value of $|\vec{x}|$ is

   a) 4 b) 7 c) 8 d) 2

6. The length of the latus rectum for the ellipse $\frac{x^2}{64} + \frac{y^2}{16} = 1$ is

   a) 2 b) 3 c) 4 d) 5

7. If two books are to be selected at random without replacement out of four books, then the number of possible selections is

   a) 4 b) 2 c) 6 d) 3

8. The slope of the normal to the curve $y = x^3 + 2x^2 + 3x - 10$ at $(-3, 2)$ is

   a) – 18 b) 18 c) – 1/18 d) 1/18

9. The order and degree of the differential equation $\sqrt{\left(\frac{dy}{dx}\right)^4 + 4} = \left(\frac{d^2y}{dx^2}\right)^6$ are respectively

   a) 2, 6 b) 2, 3 c) 1, 4 d) 2, 12

10. You have a system of three linear equations with three unknowns. If you perform Gaussian elimination and obtain the row-reduced echelon form $\left(\begin{array}{ccc|c} 1 & -2 & 4 & 6 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 3 & 0 \end{array}\right)$, then the system has

    a) a unique solution b) no solution

    c) infinitely many solutions d) finite number of solutions

11. If two like parallel forces 38 N and 86 N are acting at a distance of 6cm, then the resultant and its position are

    a) 416 N, 1.24 cm from Q b) 124 N, 4.16 cm from Q

    c) 124 N, 4.16 cm from P d) 416 N, 1.24 cm from P

    OR

    Consider the macroeconomic model:
    $G = 30$ (government expenditure), $I = 90$ (planned investment), $C = 0.8Y + 20$ (consumption) and $Y = C + G + I$ (equilibrium).
    If the government expenditure rises by 1 unit, then the change in the value of national income Y is

    a) 13 b) 10 c) 8 d) 18

GROUP B **[8 × 5 = 40]**

12. a) State De Moivre's theorem. Using it, find the square roots of $2 - 2\sqrt{3}\,i$.

    b) If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ca) = 0$ are equal, prove that either $a = 0$ or $a^3 + b^3 + c^3 - 3abc = 0$.

    [3+2]

13. a) Using mathematical induction, prove that

    $2 + 4 + 6 + 8 + \ldots + 2n = n(n + 1)$.

    b) Solve the system $x + 2y + 3z = 6$, $2x + 4y + z = 7$ and $3x + 2y + 9z = 14$ by the row–equivalent matrix method.

    [2+3]

14. a) Express $\cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5}$ in terms of $\sin^{-1}$.

b) Find the equation of the hyperbola with the focus at (–5, 0) and the vertex at (2,0).

[3+2]

15. a) For the observations of the variables X and Y, the following results are obtained $\Sigma X = 50$, $\Sigma Y = 75$, $\Sigma X^2 = 700$, $\Sigma XY = 500$, $n = 32$.

Find the equation of the line of regression of Y on X. Estimate the value of Y when $X = 25$.

b) Find the binomial distribution having mean = 12 and variance = 8.

[3+2]

16. Compute the integrals a) $\int \frac{dx}{a + b\cos x}$ $(a > b > 0)$ b) $\int \frac{2x - 11}{x^2 + x - 2}\,dx$. [3+2]

17. Write Bernoulli's equation. Solve $\frac{dy}{dx} + \frac{1}{x}y = x^2 y^6$. [1+4]

18. A small industry manufactures necklaces and bracelets. The combined number of necklaces and bracelets that it can handle per day is not more than 24. Each bracelet takes 1 hour of labour to make and each necklace takes a half hour. The total number of hours of labour available does not exceed 16. If the profit on the necklace is Rs.80 and the profit on the bracelets is 50.

a) Formulate the given problem mathematically.

b) For maximizing profit, how many of each product should be produced daily? Solve the problem by the simplex method.

[1+4]

19. a) A uniform beam, 4 m long, is supported in a horizontal position by two props which are 3 m apart, so that the beam projects one meter beyond one of the props. Show that the force on one of the props is double of that on the other.

b) A ball is projected at an angle of 30° to the horizontal and land on the surface of height 10 m which is $20\sqrt{3}$ m. away from the point of projection. Find the velocity of projection and its striking velocity on the surface. ($g = 10$ m/s^2)

[2+3]

OR

a) If the fixed cost for a good is Rs 18, the variable cost per unit is Rs 4, and the demand function is $P = 24 - 2Q$, find an expression for the profit function in terms of Q. What is the maximum profit? For what values of Q does the firm break even?

b) The demand function for a commodity is $p_d = 113 - x^2$ and the supply function is $p_s = (x + 1)^2$. Find the consumer's surplus at the equilibrium market price.

[3+2]

GROUP C [3 × 8 = 24]

20. a) An examination paper consists of 12 questions divided into two parts A and B. Part A contains 7 questions and Part B contains the remaining questions. A candidate is required to attempt 8 questions selecting at least 3 from each part. In how many ways can the candidate select the questions?

b) If $x = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \ldots$ show that $y = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$

c) Let $G = \{1, -1, i, -i\}$ where i is an imaginary unit and * stands for the binary operation of multiplication. Show that (G, *) forms a group.

[3+2+3]

21. a) Find the direction cosines l, m, n of two lines which are connected by the relations $l + m + n = 0$ and $mn - 2nl - 2lm = 0$.

b) Define vector product of two vectors. Interpret it geometrically. Find the area of the triangle determined by the vectors $3\vec{i} + 4\vec{j}$ and $-5\vec{i} + 7\vec{j}$.

[5+3]

22. a) Let $f(x) = e^{\sin x}$. Find $\frac{d}{dx} f(x)$ from first principle.

b) Find the derivative of $\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)^{nx}$

c) State L'Hospital's Rule. Using it, find the value of $\lim_{x\to 1}\left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$.

[4+2+1+1]

THE END