

For regular students (excluding technical stream's students)*Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.***Time: 3 hrs.****Full Marks: 75**Attempt **all** the questions.**Group 'A'**

11×1=11

Rewrite the correct options of each question in your answer sheet.

- What is the amplitude of a complex number i ?
A) -1 B) 90° C) 270° D) -90°
- How many three-digit numbers can be formed by using the integers 4,5,6,8 with repetition ?
A) 64 B) 27 C) 24 D) 6
- Which one of the the following is equal to $\cot^{-1} x$?
A) $\frac{1}{\cot x}$ B) $\frac{1}{\tan^{-1} x}$
C) $\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$ D) $\cos^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$
- Which one of the following expressions is a trigonometric equation ?
A) $2\sin x \cdot \cos x = \frac{2\tan x}{1+\tan^2 x}$ B) $\frac{\tan \theta + \tan 2\theta}{1 - \tan \theta \cdot \tan 2\theta} = \tan 3\theta$
C) $\sin^2 x + \cos^2 x = 1$ D) $2\cos^2 x - 1 = 0$

Contd...

- What is the area of a triangle with vertices (1,-1,0), (0, -1, 0) and (0,0,1) ?
A) $\sqrt{2}$ B) 1 C) $\frac{1}{\sqrt{2}}$ D) $\frac{1}{2}$
- In a school ground, the area is the set of all points in a plane. The sum of whose distances from two fixed points is constant. The enclosed area represented gives a geometrical form
A) circle B) parabola C) ellipse D) hyperbola
- What is the probability of getting 53 Fridays or Saturdays or both in a leap year ?
A) $\frac{1}{7}$ B) $\frac{2}{7}$ C) $\frac{3}{7}$ D) $\frac{1}{122}$
- If the degree of the differential equation $\frac{d^4 y}{dx^4} + 2\left(\frac{d^3 y}{dx^3}\right)^2 + 3\left(\frac{d^2 y}{dx^2}\right)^3 + 4\left(\frac{dy}{dx}\right)^4 + 5 = 0$ is 1, what is its order ?
A) 1 B) 2 C) 3 D) 4
- What is the point on the curve $y = 2x^2 - 4x - 3$ at which the tangent of the curve is parallel to the line $4x - y + 2 = 0$?
A) (2,-3) B) (-1, 3) C) (0, -3) D) (0, 2)
- While solving a system of three linear equations in the three variables x, y, z by using Gauss elimination method, what happens to the system if the third variable z is found to be a free variable ?
A) The system becomes consistent with infinite solutions.
B) The system becomes consistent with an unique solution.
C) The system becomes inconsistent with so solution.
D) The system cannot be solved by Gauss elimination method.
- If a constant force of 'F' N acts on a body of mass 'm' kg for 't' seconds and brings it into motion with an acceleration 'a' m/sec² and velocity Vm/sec, what will be its impulse ?
A) ma B) Fa C) Ft D) Fv

Or

If X be the output vector, D be the demand vector, A be the input coefficient matrix and I be the unit matrix then the Gontif's technology matrix is given by

- A) $I + A$ B) $I + X$ C) $I - A$ D) $I - X - D = 0$

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(3)

0081'J'

Group 'B'

8×5=40

12. The numbers 1, 2, 3, 4 ∞ are said to be natural numbers. These numbers are used to form various series. 5×1

- Write an expression for the sum of first (n+2) natural numbers.
- Write the formula for the sum of first (n+1) even natural numbers.
- Write the formula for the general term of the sequence formed by the first 'n' odd natural numbers.
- Write the formula for the sum of squares of first (n-1) natural numbers.
- Write the formula for the sum of cubes of first 'n' natural numbers.

13. If $(1+x)^n = c_0 + c_1 x + c_2 x^2 + \dots + c_n x^n$, prove that

$$c_0 c_3 + c_1 c_4 + c_2 c_5 + \dots + c_{n-3} c_n = \frac{(2n)!}{(n-3)!(n+4)!}$$

14. a) Find the value of $\sin\left(\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13}\right)$ (3)

b) Find the unit vector perpendicular to each of the vectors $\vec{a} = (1, 0, -2)$

and $\vec{b} = (2, 0, -1)$

15. The following table gives the information about the ages (in years) and corresponding weights (in kg) of six students of grades 8, 10 and 12 selected for a survey of dietary purpose.

Ages (yrs.)	13	15	14	16	18	21
Weights (in kg)	45	41	32	45	50	55

Estimate the weight of a student of age 17 years. What type of correlation is between the ages and the corresponding weights? (4+1)

16. A residential student of a school is putting the following questions to his roommates. Answer these questions.

- What is the relation between continuity and differentiability of a function at a point? (1)
- What is the first principle of differentiation? (1)
- Write any one condition where we use the derivative to find the limit of a function. (1)
- Write one difference between differentiation and integration of a function. (1)

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0081'J'

(4)

e) In the expression $\int f(x)dx = F(x) + C$, what is the relation between $f(x)$ and $F(x)$ from differentiation point of view? (1)

17. a) Find $\frac{dy}{dx}$ of $x^{\sin y} = y^{\cos x}$.

b) Integrate : $\int \frac{dx}{x(1-x)}$

18. Using Simplex method, maximise $F = 18x + 8y$ subject to

$$x + 2y \leq 25, 3x + 2y \leq 28, x, y \geq 0$$

19. a) A man is carrying a load attached at the end of a stick placed over his shoulder. Show that the pressure at his shoulder is directly proportional to the distance between the shoulder and the load. (3)

b) Two bullets are fired from a point simultaneously with the same initial velocities at the angles of 35° and 55° with the horizontal line. Show that they will strike the ground at equal distances from the point of projection.

Or

The demand function for a commodity is $P = 240 - 20Q^2$. The fixed cost of the commodities is Rs. 300 and the variable cost for each additional commodity Rs. 80.

a) Express Total Revenue (TR) and Total Cost (TC) in terms of Q (1+1)

b) Find the break even points (1)

c) Calculate the profit or loss corresponding to the number of commodities

$$Q = 10 \quad (2)$$

Group 'C'

3×8=24

20. A factory produces three articles A, B and C each of which is processed by three machines P, Q and R. The time (in. hours) required for per unit of A, B and C in the machines P, Q and R are given below.

Articles	Machines		
	P	Q	R
A	3	4	2
B	4	2	3
C	2	3	4

If each of the machine is operated for 18 hours, find the number of articles A, B and C produced daily by performing the following tasks :

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(5)

0081'J'

i) Express the above information in form of equations and solve them by Cramer's rule. (3+3)

ii) If the cost of production of each unit of the articles A,B and C are Rs. 200, Rs. 160 and Rs. 120 respectively, find the total cost by using the matrix multiplication. (2)

21. a) If $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$ be any point in space, what do the points (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) represent geometrically? Find the coordinates of the point which divides the line joining the points $(3, 4, 5)$ and $(-5, -6, -8)$ externally in the ratio $2:1$. (1+3)

b) Find the direction cosines of a line which is perpendicular to the lines with direction cosines proportional to $-3, 1, -1$ and $3, -2, -4$. Also find the angle between the latter pair of lines. (2+2)

22. Some of the ordinary differential equations of first degree and first order are given below. Identify one homogeneous and one linear differential equation and solve them. (1+1+3+3)

i) $(x^2 - y^2) dx + 2xydy = 0$ when $x = 1, y = 1$

ii) $\frac{dy}{dx} - y \tan x = e^x \sec x$ When $x = 0, y = 1$

iii) $\frac{dy}{dx} - y \tan x = -y \sec^2 x$

iv) $y^2 dx + 2xydy = 0$