

1 CHAPTER



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(1564-1642)

Rotational Dynamics

Syllabus:

Equation of angular motion, Relation between linear and angular kinematics, Kinetic energy of rotation of rigid body, Moment of inertia; Radius of gyration, Moment of inertia of a uniform rod, Torque and angular acceleration for a rigid body, Work and power in rotational motion, Angular momentum, conservation of angular momentum.



Learning Objectives

After studying this chapter, students should be able to:

- Recall equations of angular motion and compare them with equations of linear motion
- Derive the expression for rotational kinetic energy
- Describe the term moment of inertia and radius of gyration
- Find the moment of inertia of thin uniform rod rotating about its center and its one end
- Establish the relation between torque and angular acceleration of a rigid body
- Describe the work and power in rotational motion with expression
- Define angular momentum and prove the principle of conservation of angular momentum
- Solve numerical problems and conceptual questions regarding the rotational dynamics

1.1 Introduction

In translational motion, a body as a whole moves along a straight line or curved line. In this chapter, we focus our study to another type of motion in which a body turns about an axis, called rotational motion. A solid body is said to be rigid if the particles in the body are compactly arranged and the interparticle distance is not disturbed by an external force applied on it. The shape of a rigid body remains unaltered under the application of any external force. No real body is perfectly rigid. But, for all practical purposes, all the solid bodies are regarded as rigid bodies. Rigidity is the property of a rigid body by virtue of which it offers resistance to the external deforming force.

1.2 Rotation of Rigid Bodies

A rigid body can undergo both translational and rotational motion. A rigid body is said to be in translational motion if each particle of the body has same linear displacement in the equal interval of time. For example; when a bus is moving, then the passengers and the bus itself are in translational motion. A rigid body is said to be in rotational motion about a given axis if each particle of the body has same angular displacement in the equal interval of time. In rotational motion, the different particles of the rigid body have same angular velocity but different linear velocities. For example; the motion of a wheel of a moving bus about its axle is rotational motion.

1.3 Equations of Angular Motion

When a rigid body is rotating about an axis, then its each particle moves in a circular path.

Angular Displacement

The angle through which a particle rotates in a certain interval of time is called angular displacement. It is denoted by θ and is measured in radian in S.I. Unit.

Angular Velocity

The rate of change of angular displacement is called angular velocity. That is,

$$\text{Angular velocity} = \frac{\text{Angular displacement}}{\text{time taken}}$$

$$\text{or, } \omega = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{d\theta}{dt} \quad \dots (1)$$

Again, from figure 1.1,

$$d\theta = \frac{ds}{r}$$

$$\text{or, } ds = r d\theta \quad \dots (2)$$

Where, r = radius of the circular path,

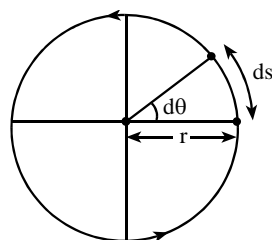
ds = linear displacement in time interval dt

and, $d\theta$ = angular displacement in time interval dt .

Now, dividing both sides of equation (2) by dt , we get

$$\frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\text{or, } v = r\omega \quad \dots (3)$$



[Fig. 1.1, Angular

where, $\frac{ds}{dt} = v$ = linear velocity, and $\frac{d\theta}{dt} = \omega$ = angular velocity

Angular velocity (ω) is measured in rad/sec. and dimensions is $[M^0L^0T^{-1}]$

Angular Acceleration

The rate of change of angular velocity is called angular acceleration.

That is, Angular acceleration = $\frac{\text{change in angular velocity}}{\text{time taken}}$

$$\text{or, } \alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{d\omega}{dt} \quad \dots (4)$$

$$\text{or, } \alpha = \frac{\omega_2 - \omega_1}{t}, \text{ where } t_2 - t_1 = t$$

$$\therefore \omega_2 = \omega_1 + \alpha t \quad \dots (5)$$

Which is similar to, $v = u + at$, in linear motion. Angular acceleration is measured in rad/sec^2 and dimensions is $[M^0L^0T^{-2}]$

Again, average angular velocity is, $\omega_{av} = \frac{\omega_1 + \omega_2}{2}$

$$\text{Also, } \omega_{av} = \frac{\theta}{t}$$

$$\therefore \frac{\theta}{t} = \frac{\omega_1 + \omega_2}{2}$$

$$\text{or, } \theta = \left(\frac{\omega_1 + \omega_2}{2} \right) t \quad \dots (6)$$

From equations (5) and (6), we have

$$\theta = \left(\frac{\omega_1 + \omega_1 + \alpha t}{2} \right) t = \left(\frac{2\omega_1 + \alpha t}{2} \right) t$$

$$\text{or, } \theta = \left(\omega_1 + \frac{1}{2} \alpha t \right) t$$

$$\therefore \theta = \omega_1 t + \frac{1}{2} \alpha t^2 \quad \dots (7)$$

This equation is similar to, $s = ut + \frac{1}{2} at^2$, in linear motion.

Now, squaring equation (5), we get

$$\omega_2^2 = \omega_1^2 + 2\omega_1 \alpha t + \alpha^2 t^2$$

$$\text{or, } \omega_2^2 = \omega_1^2 + 2\alpha \left(\omega_1 t + \frac{1}{2} \alpha t^2 \right)$$

$$\therefore \omega_2^2 = \omega_1^2 + 2\alpha \theta \quad \dots (8) \text{ [Using equation (7)]}$$

This equation is similar to, $v^2 = u^2 + 2as$, in linear motion.

1.4 Kinetic Energy of a Rotating Body

Let us consider a rigid body having mass M is rotating about an axis YY' with constant angular velocity ω as shown in figure 1.2. Let the body consists of a number of particles having masses $m_1, m_2, m_3, \dots, m_n$ which are at perpendicular distances $r_1, r_2, r_3, \dots, r_n$ respectively from the axis of rotation. All of these n particles have same angular velocity but different linear velocities. Let their respective linear velocities be $v_1, v_2, v_3, \dots, v_n$. Then, $v_1 = r_1\omega, v_2 = r_2\omega, v_3 = r_3\omega, \dots, v_n = r_n\omega$

The rotational kinetic energy of particle m_1 is,

$$K.E_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 (r_1 \omega)^2 = \frac{1}{2} m_1 r_1^2 \omega^2$$

Similarly, the rotational kinetic energy of particles of masses m_2, m_3, \dots, m_n are

$$K.E_2 = \frac{1}{2} m_2 r_2^2 \omega^2, K.E_3 = \frac{1}{2} m_3 r_3^2 \omega^2, \dots, K.E_n = \frac{1}{2} m_n r_n^2 \omega^2 \text{ respectively.}$$

Now, the rotational kinetic energy of a rigid body is equal to the sum of the kinetic energies of the particles of the body. Therefore,

$$K.E_{\text{rot}} = K.E_1 + K.E_2 + K.E_3 + \dots + K.E_n$$

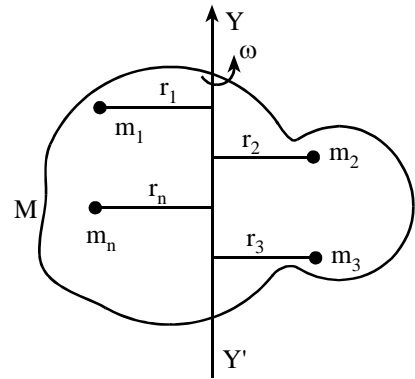
$$\text{or, } K.E_{\text{rot}} = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2 + \dots + \frac{1}{2} m_n r_n^2 \omega^2$$

$$\text{or, } K.E_{\text{rot}} = \frac{1}{2} (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2) \omega^2$$

$$\text{or, } K.E_{\text{rot}} = \frac{1}{2} (\Sigma m r^2) \omega^2$$

$$\therefore K.E_{\text{rot}} = \frac{1}{2} I \omega^2$$

Where, $m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2 = \Sigma m r^2 = I$, is the moment of inertia of the body about the axis of rotation YY' . The above equation is analogous to $K.E. = \frac{1}{2} m v^2$ is translational motion.

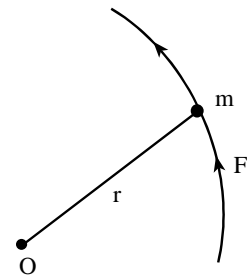


[Fig. 1.2, Kinetic Energy of rotation]

1.5 Moment of Inertia (Rotational Inertia)

The inertia of a body in rotational motion is called **moment of inertia**. It is the inability of a body to change its state of rest or state of uniform rotational motion by itself. A body rotating about an axis has a tendency to be rotating even if a stopping torque is applied to it. Such a property of a rotating body is called **rotational inertia or moment of inertia**. For example, a rotating fan doesn't stop immediately even if the switch is put off due to rotational inertia.

Suppose, a particle having mass ' m ' is rotating about an axis passing through a point ' O ' which is at a distance r from the particle as shown in figure 1.3. If F be the force applied on the particle, then



[Fig. 1.3 : Moment of inertia]

$$F = ma$$

Where 'a' is the tangential acceleration of the particle

$$\text{Also, } a = r\alpha$$

Where, α is the angular acceleration of the particle. Now, torque on the particle due to this force F is

$$\tau = r \times F = r \times ma = r \times m(r\alpha)$$

$$\therefore \tau = (mr^2)\alpha \quad \dots (1)$$

For a linear motion, we have,

Force = linear inertia (mass) \times linear acceleration.

For a rotational motion, we must have

Torque = Rotational inertia (moment of inertia) \times angular acceleration

$$\therefore \tau = I\alpha \quad \dots (2)$$

Comparing equations (1) and (2), we get

$$I = mr^2 \quad \dots (3)$$

Hence, the moment of inertia (I) of a particle about an axis is measured as the product of its mass and square of its distance from the axis of rotation. In SI unit, I is measured in kgm^2 and in CGS unit, it is expressed in gm cm^2 . The dimensional formula, of I is, $I = M \times L^2 = [ML^2 T^0]$.

1.6 Moment of Inertia of a Rigid Body

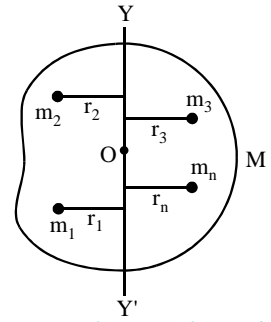
Consider a rigid body having mass M is rotating about an axis YOY' with constant angular velocity ω as shown in figure 1.4. Let the rigid body consists of n particles of masses m_1, m_2, \dots, m_n which are at perpendicular distances r_1, r_2, \dots, r_n respectively from the axis YOY'. Then the moment of inertia of these n particles about the axis YOY' are given by $I_1 = m_1 r_1^2, I_2 = m_2 r_2^2, \dots, I_n = m_n r_n^2$ respectively. Now, the moment of inertia (I) of the rigid body about the axis YOY' is equal to the sum of moments of inertia of these all particles. Therefore,

$$I = I_1 + I_2 + \dots + I_n$$

$$\text{or, } I = m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2$$

$$\therefore I = \sum_{i=1}^n m_i r_i^2$$

Where m_i is the mass of i^{th} particle of the body which is at a distance r_i from the axis YOY'. Thus, the moment of inertia (I) of a rigid body about an axis is the sum of the products of the masses of its particles and the squares of their respective perpendicular distances from the axis of rotation. The moment of inertia (I) of a body depends on the mass of the body and the distribution of its particles. The moment of inertia of a body is not a fixed quantity. It has different values for different axis of rotation for a body. The moment of inertia (I) plays the same role in rotational motion as the mass (m) plays in the linear motion.



[Fig. 1.4, Moment of inertia of a rigid body]

1.7 Moment of Inertia of a Uniform Rod

A. About an axis through its centre and perpendicular to its length

Suppose AB be a thin uniform rod having mass M and length l as shown in fig 1.5. Let YOY' be an axis passing through centre O of the rod and perpendicular to its length about which the M.I. has to be determined. Consider a small length dx of the rod which lies at a distance x from the centre O of the rod and has mass dm .

Here, mass per unit length of the rod = $\frac{M}{l}$.

Mass of element dx of the rod, $dm = \left(\frac{M}{l}\right) dx$

The small moment of inertia of mass dm about the axis YOY' is

$$dI = (dm) x^2 = \left(\frac{M}{l} dx\right) x^2$$

$$\therefore dI = \left(\frac{M}{l}\right) x^2 dx \quad \dots (1)$$

The total moment of inertia of the rod AB about the axis YOY' is obtained by integrating equation (1) from the limit $x = -\frac{l}{2}$ to $x = \frac{l}{2}$ i.e.,

$$I = \int_{-l/2}^{l/2} dI = \int_{-l/2}^{l/2} \left(\frac{M}{l}\right) x^2 dx \quad [\text{Using equation (1)}]$$

$$\text{or, } I = \frac{M}{l} \int_{-l/2}^{l/2} x^2 dx = \frac{M}{l} \left[\frac{x^3}{3} \right]_{-l/2}^{l/2} = \frac{M}{3l} \left[\left(\frac{l}{2}\right)^3 - \left(-\frac{l}{2}\right)^3 \right] = \frac{M}{3l} \left[\frac{l^3}{8} + \frac{l^3}{8} \right]$$

$$\therefore I = \frac{Ml^2}{12} \quad \dots (2)$$

B. About an axis through its one end and perpendicular to its length

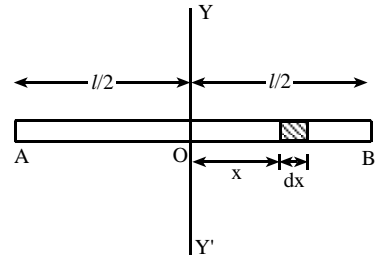
In this case, the axis lies at one end, say A, and is perpendicular to its length as shown in figure 1.6. Now, the M.I. can be obtained by integrating equation (1) from limit $x = 0$ to $x = l$, i.e.

$$I = \int_0^l dI = \int_0^l \left(\frac{M}{l}\right) x^2 dx \quad [\text{Using equation (1)}]$$

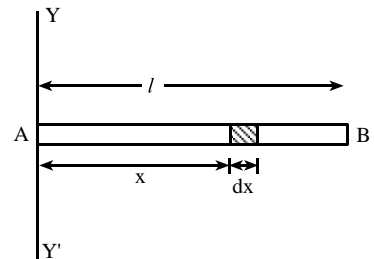
$$\text{or, } I = \frac{M}{l} \int_0^l x^2 dx = \frac{M}{l} \left[\frac{x^3}{3} \right]_0^l = \frac{M}{3l} [l^3 - 0^3]$$

$$\therefore I = \frac{Ml^2}{3} \quad \dots (3)$$

Equations (2) and (3) show that the M.I. of a thin uniform rod about an axis passing through its one end and perpendicular to its length is four times greater than that of the rod about an axis passing through its centre and perpendicular to its length.



[Fig. 1.5, M.I. of a uniform rod.]



[Fig. 1.6, M.I. of a uniform rod.]

1.8 Moment of Inertia of Various Bodies

S.N.	Body	Moment of Inertia	Axis
1.	Rectangular plate	$I = \frac{M(l^2 + b^2)}{12}$	Through its centre and perpendicular to its plane
2.	Circular ring	$I = MR^2$	Through its centre and perpendicular to its plane
3.	Circular disc	$I = \frac{MR^2}{2}$	Through its centre and perpendicular to its plane
4.	Solid cylinder	$I = \frac{MR^2}{2}$	Through its axis of symmetry
5.	Hollow cylinder	$I = MR^2$	Through its axis of symmetry
6.	Solid sphere	$I = \frac{2}{5} MR^2$	About its any diameter
7.	Hollow sphere	$I = \frac{2}{3} MR^2$	About its any diameter

1.9 Radius of Gyration

The radius of gyration of a body about a given axis of rotation is defined as the distance from the axis at which the whole mass of the body is supposed to be concentrated so that the moment of inertia of the body about the axis of rotation remains the same. It is denoted by K .

Suppose, a body consists of n particles, each of mass m . Thus, by definition of moment of inertia

$$I = \sum mr^2$$

$$\text{or } I = m [r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2]$$

$$\text{or, } I = mn \left[\frac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n} \right]$$

$$\therefore I = MK^2 \quad \dots (1)$$

Where, $mn = M = \text{Total mass of the body}$

$$\text{and, } K^2 = \frac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n}$$

$$\therefore K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n}} \quad \dots (2)$$

Thus, the radius of gyration of a body about a given axis can also be defined as the **root mean square distances** of all the particles of the body from the axis of rotation.

Radius of Gyration of a Thin Rod

The M.I. of a thin rod about an axis through its centre and perpendicular to its length is,

$$I = \frac{ML^2}{12} \quad \dots (3)$$

If K be the radius of gyration of the rod about the axis, then,

$$I = MK^2 \quad \dots (4)$$

From equations (3) and (4), we get

$$MK^2 = \frac{Ml^2}{12}$$

$$\therefore K = \frac{l}{\sqrt{12}}$$

If the axis of rotation passes through one end and perpendicular to the length of the rod, then

$$MK^2 = \frac{Ml^2}{3}$$

$$\therefore K = \frac{l}{\sqrt{3}}$$

Similarly, Radius of Gyration of various bodies are given in the following table:

S.N.	Body	Moment of Inertia(I)	Radius of Gyration(K)	Axis
1.	Rectangular plate	$MK^2 = \frac{M(l^2 + b^2)}{12}$	$K = \sqrt{\frac{l^2 + b^2}{12}}$	Through its centre and perpendicular to its plane.
2.	Circular ring	$MK^2 = MR^2$	$K = R$	Through its centre and perpendicular to its plane.
3.	Circular disc.	$MK^2 = \frac{MR^2}{2}$	$K = \frac{R}{\sqrt{2}}$	Through its centre and perpendicular to its plane.
4.	Solid cylinder	$MK^2 = \frac{MR^2}{2}$	$K = \frac{R}{\sqrt{2}}$	Through its axis of symmetry
5.	Hollow cylinder	$MK^2 = MR^2$	$K = R$	Through its axis of symmetry
6.	Solid sphere	$MK^2 = \frac{2}{5} MR^2$	$K = \sqrt{\frac{2}{5}} R$	About its any diameter
7.	Hollow sphere	$MK^2 = \frac{2}{3} MR^2$	$K = \sqrt{\frac{2}{3}} R$	About its any diameter

1.10 Torque or Moment of a Force

The turning effect of a force on a body is called **torque or moment of the force**. It is denoted by τ . The torque acting on a rotating body is measured as the product of the force and perpendicular distance of the force from the axis of rotation. Mathematically,

Torque = Force \times perpendicular distance of the force from the axis of rotation.

$$\text{or, } \tau = Fr$$

$$\therefore \tau = rF \quad \dots (1)$$

It is a vector quantity. Its SI unit is Newton meter (Nm). In vector form, torque is expressed as,

$$\vec{\tau} = \vec{r} \times \vec{F} \quad \dots (2)$$

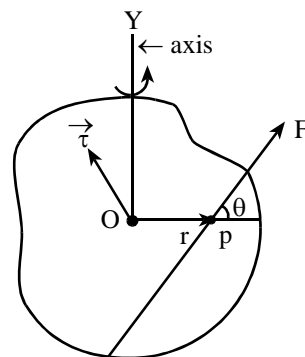
The direction of $\vec{\tau}$ is perpendicular to the plane containing \vec{r} and \vec{F} . The torque may be clockwise torque or anticlockwise torque. Its dimensional formula is,

$$\tau = rF = L \times MLT^{-2} = [ML^2T^{-2}].$$

Equation (2) can be written as,

$$\vec{\tau} = \vec{r} \times \vec{F} = rF \sin\theta \hat{n} \quad \dots (3)$$

Where, θ is the angle between \vec{r} and \vec{F} , and \hat{n} is unit vector along the direction of $\vec{\tau}$.



[Fig. 1.7, Torque or moment of force]

When $\theta = 0^\circ$, then $\tau = Fr \sin 0^\circ = 0 = \text{minimum}$

When $\theta = 90^\circ$, then $\tau = Fr \sin 90^\circ = Fr = \text{maximum}$

Relation between Torque, Angular Acceleration and Moment of Inertia

Suppose a rigid body is rotating about an axis YY' with angular velocity ω as shown figure 1.8. Suppose the body consists of n particles having masses $m_1, m_2, m_3, \dots, m_n$ which are at a respective distances $r_1, r_2, r_3, \dots, r_n$ from the axis of rotation. Let τ be the external torque applied to the body. The torque τ will produce constant angular acceleration α on each particle but different linear accelerations. If $a_1, a_2, a_3, \dots, a_n$ be the respective linear accelerations produced on n particles, then

$$a_1 = r_1\alpha, a_2 = r_2\alpha, a_3 = r_3\alpha, \dots, a_n = r_n\alpha$$

The forces acting on n particles are,

$$F_1 = m_1 a_1, F_2 = m_2 a_2, \dots, F_n = m_n a_n \text{ respectively.}$$

Similarly, the torques due to these n forces are

$$\tau_1 = F_1 r_1, \tau_2 = F_2 r_2, \dots, \tau_n = F_n r_n \text{ respectively.}$$

Since, total torque acting on the body is equal to the sum of the individual torques acting on the n particles of the body. Therefore, total torque τ is,

$$\begin{aligned} \tau &= \tau_1 + \tau_2 + \tau_3 + \dots + \tau_n \\ &= F_1 r_1 + F_2 r_2 + F_3 r_3 + \dots + F_n r_n \\ &= (m_1 a_1) r_1 + (m_2 a_2) r_2 + (m_3 a_3) r_3 + \dots + (m_n a_n) r_n \\ &= (m_1 r_1 \alpha) r_1 + (m_2 r_2 \alpha) r_2 + (m_3 r_3 \alpha) r_3 + \dots + (m_n r_n \alpha) r_n \\ &= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2) \alpha = (\Sigma m r^2) \alpha \\ \therefore \tau &= I \alpha \end{aligned} \quad \dots (1)$$

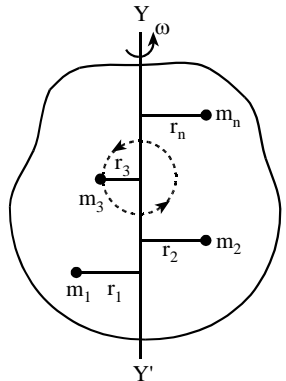
$$\text{Where, } I = \Sigma m r^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2$$

is the moment of inertia of the body about the axis of rotation YY' . Equation (1) gives the relation between torque (τ), angular acceleration (α) and moment of inertia (I).

If $\alpha = 1$, then $\tau = I \times 1$

$$\therefore I = \tau$$

So, the M.I. of a body about a given axis is equal to the external torque required to produce unit angular acceleration in the body about the axis.



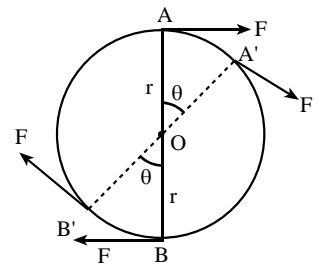
[Fig. 1.8, Rotating body]

1.11 Work Done by a Couple and Power in Rotational Motion

Two equal and unlike parallel forces acting at two different points of a rigid body forms a couple. A couple always produces the rotational motion on a body. The torque due to a couple is,

$\tau = \text{Either of the forces of a couple} \times \text{perpendicular distance between the two forces of the couple}$

Let two equal and unlike parallel forces, each of magnitude F act on a wheel at two points A and B tangentially such that AB is the diameter of the wheel. Let the wheel rotates through an angle θ and the points A



[Fig. 1.9 : Rotating wheel]

and B get shifted to new positions A' and B' respectively. The torque due to the couple is,

$$\tau = F \times 2r \quad \dots (1)$$

Where, r is the radius of the wheel

Now, work done by each force of the couple is,

$$= \text{Force} \times \text{distance moved}$$

$$= F \times AA' \text{ (or } BB')$$

$$= F \times r\theta; [\text{since } AA' = BB' = r\theta \text{ and } \theta \text{ is in radian.}]$$

The total work done by the two forces of the couple is given by

$$W = F \times r\theta + F \times r\theta$$

$$\text{or, } W = 2F \times r\theta$$

$$\text{or, } W = (F \times 2r)\theta$$

$$\therefore W = \tau\theta \dots (2) \quad [\text{using equation (1)}]$$

Which is similar to, $W = FS$, in translational motion.

Again, differentiating equation (2) with respect to time t , we get.

$$\frac{dW}{dt} = \frac{d}{dt}(\tau\theta) = \tau \frac{d\theta}{dt}$$

$$\therefore P = \tau\omega \quad \dots (3)$$

Where, $\frac{dW}{dt} = P = \text{power developed}$ and $\frac{d\theta}{dt} = \omega = \text{angular velocity}$

Equation (3) is similar to, $P = F.v$, in translational motion.

1.12 Angular Momentum

The moment of linear momentum of a body is called angular momentum. It is denoted by L . A rotating body possesses angular momentum. It is measured as the product of the linear momentum of a body and the perpendicular distance between the body and the axis of rotation.

Suppose a body having mass m is revolving around a circle of radius r with speed v about an axis passing through the centre O as shown in figure 1.10. Then angular momentum of the body is,

$L = \text{Linear momentum} \times \text{perpendicular distance between the body and axis of rotation}$

$$\text{or, } L = Pr$$

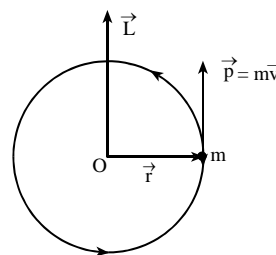
$$\text{or, } L = mvr \quad \dots (1)$$

If ω be the angular velocity of the body, then, $v = r\omega$

$$\text{So, } L = m(r\omega)r$$

$$\text{or, } L = mr^2\omega \quad \dots (2)$$

Equations (1) and (2) are the expressions for the angular momentum of the body. The S.I. unit of angular momentum is $\text{kgm}^2\text{s}^{-1}$ and C.G.S. unit is $\text{gcm}^2\text{s}^{-1}$. The dimensional formula is $[\text{ML}^2\text{T}^{-1}]$. It is a vector quantity. In vector form,



[Fig. 1.10, Body revolving around a circle.]

$$\vec{L} = \vec{r} \times \vec{P} = rp \sin\theta \hat{n} \quad \dots (3)$$

Where θ is the angle between \vec{r} and \vec{P} , and \hat{n} is the unit vector along direction of \vec{L} . The direction of \vec{L} is perpendicular to both \vec{r} and \vec{P} as shown in figure 1.10. The magnitude of angular momentum is,

$$L = rP \sin\theta \quad \dots (4)$$

When $\theta = 0^\circ$, then $L = rP \sin 0^\circ = 0$. In this case, there is no rotational effect on the body.

When $\theta = 90^\circ$, then $L = rP \sin 90^\circ = rP$. In this case, there is maximum rotational effect on the body.

Relation between Angular Momentum and Moment of Inertia

Consider a rigid body having mass M is rotating about an axis YY' with constant angular velocity ω as shown in figure 1.11. Let the body consists of n particles having masses $m_1, m_2, m_3, \dots, m_n$ which are at respective perpendicular distances $r_1, r_2, r_3, \dots, r_n$ from the axis of rotation. Here, all the particles of the body have same angular velocity ω but different linear velocities. Let $v_1, v_2, v_3, \dots, v_n$ be their respective linear velocities. Then $v_1 = r_1\omega, v_2 = r_2\omega, v_3 = r_3\omega, \dots, v_n = r_n\omega$.

The magnitude of angular momentum of particle of mass m_1 is given by, $L_1 = m_1 v_1 r_1 = m_1 (r_1 \omega) r_1 = m_1 r_1^2 \omega$. Similarly, if L_2, L_3, \dots, L_n be the magnitudes of angular momentum of particles of masses m_2, m_3, \dots, m_n respectively, then, $L_2 = m_2 r_2^2 \omega, L_3 = m_3 r_3^2 \omega, \dots, L_n = m_n r_n^2 \omega$. Now, the total angular momentum of the body about the axis YY' is equal to the sum of the angular momenta of all the n particles of the body about that axis. Thus,

$$\begin{aligned} L &= L_1 + L_2 + L_3 + \dots + L_n \\ &= m_1 r_1^2 \omega + m_2 r_2^2 \omega + m_3 r_3^2 \omega + \dots + m_n r_n^2 \omega \\ &= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2) \omega \\ &= (\Sigma m r^2) \omega \end{aligned}$$

$$\therefore L = I \omega$$

Where, $\Sigma m r^2 = I$, is the moment of inertia of the rigid body about the axis of rotation YY' . Above equation $L = I\omega$ is similar to $P = mv$ is translational motion. Thus, the angular momentum in rotational motion is analogous to linear momentum in translational motion.

Relation between Angular Momentum and Torque

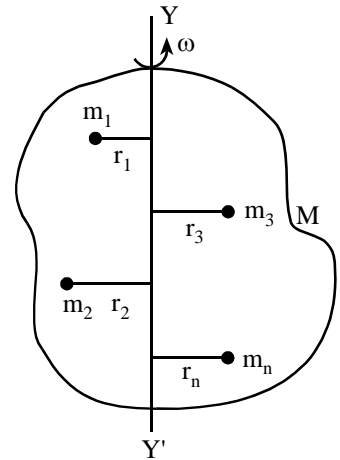
The angular momentum of a rigid body having moment of inertia I and rotating with angular velocity ω about an axis of rotation is,

$$L = I\omega \quad \dots (1)$$

Differentiating both sides w.r.t. time t , we get

$$\frac{dL}{dt} = \frac{d}{dt} (I\omega)$$

The moment of inertia I of a body about a given axis of rotation is constant.



[Fig. 1.11, Rotating rigid body]

$$\therefore \frac{dL}{dt} = I \frac{d\omega}{dt}$$

$$\text{or, } \frac{dL}{dt} = I\alpha \quad \dots (2)$$

Where, $\frac{d\omega}{dt} = \alpha$ is the angular acceleration of the body. Also, the torque acting on the body is

$$\tau = I\alpha \quad \dots (3)$$

From equations (2) and (3), we get

$$\tau = \frac{dL}{dt} \quad \dots (4)$$

Hence, the torque acting on a body is equal to the rate of change of angular momentum of the body.

Equation (4) in rotational motion is analogous to $F = \frac{dP}{dt}$ in translational motion.

1.13 Principle of Conservation of Angular Momentum

It states that if no external torque acts on a system, then total angular momentum of the system remains conserved. That is, $I\omega = \text{constant}$

Where, I is the moment of inertia of a body about a given axis of rotation and ω is its angular velocity.

Proof: We have, the torque acting on a body is,

$$\tau = \frac{dL}{dt}$$

If external torque acting is zero i.e. $\tau = 0$, then,

$$\frac{dL}{dt} = 0$$

$$\text{or, } dL = 0$$

Integrating this equation, we get

$$\int dL = \text{constant}$$

$$\text{or, } L = \text{constant}$$

$$\therefore I\omega = \text{constant} \quad [\text{Since, } L = I\omega]$$

This is the principle of conservation of angular momentum. If I_1 and ω_1 be the initial values of moment of inertia and angular velocity respectively, and I_2 and ω_2 be their respective final values, then in general

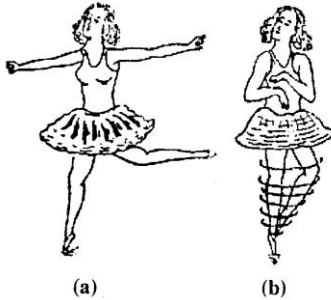
$$I_1\omega_1 = I_2\omega_2$$

Examples of Conservation of Angular Momentum

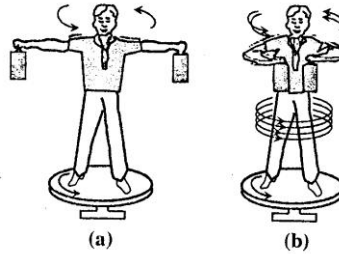
- 1. Motion of a Planet around the Sun:** The motion of a planet around the sun in elliptical orbit follows the angular momentum conservation principle. When the planet is far from the sun, its moment of inertia about the axis of rotation increases and its angular velocity ω decreases keeping $I\omega = \text{constant}$. Again, when the planet approaches near to the sun, then I decreases and ω increases keeping $I\omega = \text{constant}$.
- 2. Earth-Meteorite system:** The effective mass of earth increases when a meteorite falls on its surface. This increases moment of inertia of the earth. From angular momentum conservation principle, $I\omega$

= constant. So, ω should decrease. So, the earth should slightly slow down in its speed of rotation. But, due to very small change in speed of rotation, we don't experience it.

3. **Ballet dancing:** A ballet dancer or ice-skater uses angular momentum conservation principle to change rate of spin during the performance as shown in fig 1.12. When she stretches her arms (hands and legs), the moment of inertia increases and hence angular velocity decreases keeping $I\omega = \text{constant}$. Then the dancer decreases the spinning rate. To increase the rate of spin, she folds her arms (hands and legs) decreasing moment of inertia and hence angular velocity increases, keeping $I\omega = \text{constant}$.



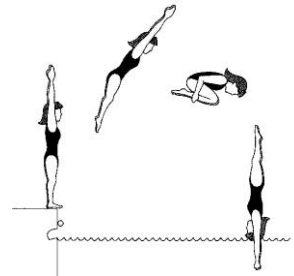
[Fig. 1.12, Ballet dancing]



[Fig. 1.13, Rotating turntable]

4. **Motion on a Rotating Turntable:** By using angular momentum conservation principle, a man standing on a rotating turntable and holding weights on his hands, can rotate with a desired angular velocity. When he draws his hands close to the chest, moment of inertia decreases and hence angular velocity increases keeping $I\omega = \text{constant}$. On the other hand, when he stretches his hands, the moment of inertia increases and hence angular velocity decreases, keeping $I\omega = \text{constant}$ as shown in figure 1.13.

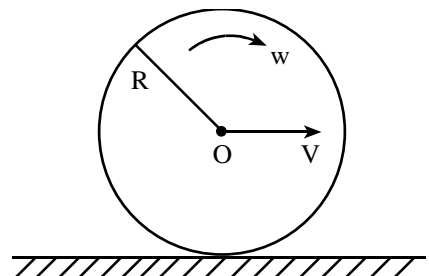
5. **A Diver diving into a Swimming Pool:** A diver diving into a swimming pool can increase the number of loops in air by using angular momentum conservation principle as shown in figure 1.14. During jumping, from a springboard, the diver curls his body by rolling his arms (hands and legs) as shown in figure. This decreases the moment of inertia and hence the angular velocity increases, keeping $I\omega = \text{constant}$. This increase in angular velocity causes the diver to have more number of loops in air.



[Fig. 1.14, Diving]

1.14 Kinetic Energy of a Rolling Body

When a body rolls, it rotates about a horizontal axis and its centre of mass moves linearly. Therefore, a rolling body possesses two types of kinetic energy translational kinetic energy and rotational kinetic energy. Suppose, a body having mass M and radius R is rolling without slipping on a horizontal plane as shown in figure 1.15. Let ω be the constant angular velocity of the body and V be the linear velocity of the centre of mass of the body. If I be the moment of inertia of the



[Fig. 1.15, K.E. of rolling body]

body about the axis of rotation, then,

K.E. of rolling body = K.E. of rotation + K.E. of translation.

$$\text{or, } K.E_T = K.E_{\text{rot}} + K.E_{\text{trans}}$$

$$\text{or, } K.E_T = \frac{1}{2} I \omega^2 + \frac{1}{2} M V^2 \quad \dots (1)$$

If K be the radius of gyration of the rolling body about the axis of rotation, then

$$I = M K^2$$

$$\therefore K.E_T = \frac{1}{2} (M K^2) \omega^2 + \frac{1}{2} M V^2$$

$$\text{or, } K.E_T = \frac{1}{2} M K^2 \left(\frac{V}{R} \right)^2 + \frac{1}{2} M V^2$$

$$\therefore K.E_T = \frac{1}{2} M V^2 \left(\frac{K^2}{R^2} + 1 \right) \quad \dots (2)$$

$$\text{Also, } K.E_T = \frac{1}{2} M (R \omega)^2 \left(\frac{K^2}{R^2} + 1 \right)$$

$$\therefore K.E_t = \frac{1}{2} M \omega^2 (K^2 + R^2) \quad \dots (3)$$

Equations (1), (2) and (3) are the expressions for kinetic energy of a rigid body rolling on a horizontal plane without slipping.

For a circular ring, $I = M R^2$. Also, $I = M K^2$

$$\therefore M K^2 = M R^2 \Rightarrow \frac{K^2}{R^2} = 1$$

$$\text{From equation (2), } K.E_T = \frac{1}{2} M V^2 (1 + 1) = M V^2$$

Similarly, the K.E. of different shaped rolling bodies are as shown in table below:

S.N.	Body	Value of $\frac{K^2}{R^2}$	$K.E_T = \frac{1}{2} M V^2 \left(\frac{K^2}{R^2} + 1 \right)$
1.	Circular disc	$\frac{1}{2}$	$\frac{3}{4} M V^2$
2.	Solid cylinder	$\frac{1}{2}$	$\frac{3}{4} M V^2$
3.	Hollow Sphere	1	$M V^2$
4.	Solid Sphere	$\frac{2}{5}$	$\frac{7}{10} M V^2$
5.	Hollow sphere	$\frac{2}{3}$	$\frac{5}{6} M V^2$
6.	Circular ring	1	$M V^2$

1.15 Acceleration of a Body Rolling Down an Inclined Plane

Let us consider a rigid body having circular symmetry (sphere, disc, cylinder etc.) of mass M and radius R is rolling down along an inclined plane having angle of inclination θ to the horizontal as shown in figure 1.16. Let V be the velocity acquired by the body after rolling down the inclined plane of height h , then the kinetic energy gained by the body is

$$K.E_T = \frac{1}{2} MV^2 \left(\frac{K^2}{R^2} + 1 \right) \quad \dots (1)$$

Where, K is the radius of gyration of the body. Also, the rigid body loses its potential energy as it rolls down the inclined plane. So,

Loss in P.E. of the body = Mgh ... (2)

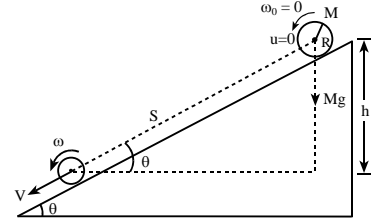
Since total mechanical energy remains conserved. So,

Loss in P.E. = Gain in K.E.

$$\text{or, } Mgh = \frac{1}{2} MV^2 \left(\frac{K^2}{R^2} + 1 \right)$$

$$\text{or, } 2gh = V^2 \left(\frac{K^2}{R^2} + 1 \right)$$

$$\therefore V^2 = \frac{2gh}{\left(\frac{K^2}{R^2} + 1 \right)} \quad \dots (3)$$



[Fig. 1.16, Acceleration of a body rolling down an inclined plane]

From figure 1.16, $\sin\theta = \frac{h}{s} \Rightarrow h = s \sin\theta$... (4)

Where, S is the distance moved along the inclined plane.

From equations (3) and (4), we have

$$V^2 = \frac{2g s \sin\theta}{\left(\frac{K^2}{R^2} + 1 \right)} \quad \dots (5)$$

If ' a ' be the linear acceleration of the rolling body down the inclined plane, then using,

$$V^2 = u^2 + 2as.$$

Since, initial velocity, $u = 0$

$$\therefore V^2 = 2as \quad \dots (6)$$

From equations (5) and (6), we get

$$2as = \frac{2g s \sin\theta}{\left(\frac{K^2}{R^2} + 1 \right)}$$

$$\therefore a = \frac{g \sin\theta}{\left(\frac{K^2}{R^2} + 1 \right)} \quad \dots (7)$$

This is the general expression for acceleration of a rigid body rolling down an inclined plane. Its value depends on the ratio of $\frac{K^2}{R^2}$ and the angle of inclination θ of the inclined plane. For a circular ring,

$$I = MK^2 = MR^2 \Rightarrow \frac{K^2}{R^2} = 1$$

$$\therefore a = \frac{g \sin\theta}{1 + 1} = \frac{1}{2} g \sin\theta$$

Similarly, the acceleration of different symmetrical bodies rolling down an inclined plane are as shown in following table:

S.N.	Body	Value of $\frac{K^2}{R^2}$	Acceleration, $a = \frac{g \sin \theta}{\left(\frac{K^2}{R^2} + 1\right)}$
1.	Circular disc	$\frac{1}{2}$	$\frac{2}{3} g \sin \theta$
2.	Solid cylinder	$\frac{1}{2}$	$\frac{2}{3} g \sin \theta$
3.	Hollow cylinder	1	$\frac{1}{2} g \sin \theta$
4.	Solid sphere	$\frac{2}{5}$	$\frac{5}{7} g \sin \theta$
5.	Hollow sphere	$\frac{2}{3}$	$\frac{3}{5} g \sin \theta$
6.	Circular ring	1	$\frac{1}{2} g \sin \theta$

1.16 Relation Between Translational and Rotational Quantities

The translational quantities and rotational quantities of a rotating body are related to each other. The relationship between the various parameters appearing in translational and rotational motions can be summarized as in the table shown below.

S.N.	Translational Motion	S.N.	Rotational Motion	Relation
1.	Linear displacement = S	1.	Angular displacement = θ	$S = r\theta$
2.	Linear velocity, $V = \frac{dS}{dt}$	2.	Angular velocity; $\omega = \frac{d\theta}{dt}$	$V = r\omega$
3.	Linear acceleration $a = \frac{dv}{dt} = \frac{d^2S}{dt^2}$	3.	Angular acceleration $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$	$a = r\alpha$
4.	Mass = m	4.	Moment of inertia = I	$I = mr^2$
5.	Linear momentum, $P = mV$	5.	Angular momentum $L = I\omega$	$L = rP$
6.	Force, $F = \frac{dP}{dt} = ma$	6.	Torque, $\tau = \frac{dL}{dt} = I \frac{d\omega}{dt} = I\alpha$	$\tau = rF$
7.	Work done by a force, $W = FS$	7.	Work done by a torque, $W = \tau\theta$	—
8.	Power, $P = FV$	8.	Power, $P = \tau\omega$	—
9.	Translational K.E. = $\frac{1}{2}mV^2$	9.	Rotational K.E. = $\frac{1}{2}I\omega^2$	—
10.	Initial linear velocity u	10.	Initial angular velocity ω_0	$u = r\omega_0$
11.	Equation of translational motion (i) $S = ut$ (ii) $V = u + at$ (iii) $s = ut + \frac{1}{2}at^2$ (iv) $V^2 = u^2 + 2as$	11.	Equations of rotation motion (i) $\theta = \omega t$ (ii) $\omega = \omega_0 + \alpha t$ (iii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ (iv) $\omega^2 = \omega_0^2 + 2\alpha\theta$	—

Boost for Objectives

- ☞ A rigid body is a system of particles in which distance between any two particles does not change under the influence of external forces.
- ☞ Moment of inertia depends on mass and distribution of mass of body about the axis of rotation.
- ☞ The spokes are fitted in cycle wheel to increase moment of inertia so that the cycle runs smoother and steadier.
- ☞ Fly wheel is an important part of engine. It helps the engine in keeping the speed uniform.
- ☞ A couple produces purely rotational motion.

- ☞ Ice skaters use the principle of conservation of angular momentum.
- ☞ If body shrinks, moment of inertia decreases.
- ☞ A dancer on ice spins faster, when she folds her arms, then M.I decreases, angular velocity is increased, K.E increases and angular momentum is conserved ($I_1\omega_1 = I_2\omega_2$)
- ☞ When a mass is rotating in plane about a fixed point, its angular momentum is directed along the axis of rotation
- ☞ If polar ice on the earth gets melted then length of the day on the earth gets increased.
- ☞ When a solid cylinder, a solid sphere and a hollow sphere are each released from same height on an inclined plane, then solid sphere reach the bottom at first.
- ☞ A body can't roll on an inclined plane in the absence of friction i.e. on smooth inclined plane.
- ☞ When a body starts to roll on an inclined plane, its potential energy is converted into translational and rotational kinetic energy.
- ☞ Position of centre of mass is independent of the reference frame. It depends only on masses of the particles and their relative positions.



Short Questions with Answers

1. What is the counterpart of the mass in rotational motion?

- ☞ We know, in linear motion,
Force = mass \times linear acceleration
or, $F = ma$... (1)
In rotational motion,
Torque = (moment of inertia) \times angular acceleration
or, $\tau = I\alpha$... (2)
From equations (1) and (2), we see that in rotational motion, moment of inertia is the counter part of mass. Moment of inertia plays the same role in rotational motion as mass plays in translational (linear) motion.

2. What is the counterpart of force in rotational motion?

- ☞ From equations (1) and (2) in question number 1, we see that in rotational motion, torque is the counter part of force. Torque plays the same role in rotational motion as force plays in translational (linear) motion.

3. A fan with blades takes longer time to come to rest than without the blades, why? [HSEB 2067 S 2051]

- ☞ A fan with blades has more moment of inertia than a fan without blades. Thus, the inertia of motion is more for a fan with blades than a fan without blades. Hence, the fan with blades takes longer time to come to rest than without the blades.

4. Suppose that only two external forces act on a rigid body and the two forces are equal in magnitude but opposite in direction. Under what conditions will the body rotate? [HSEB 2054]

- ☞ Suppose, two equal and opposite forces are acting on a body, then these two forces will rotate the body if their lines of action are different. In such case, the two forces form a couple.

5. A small girl standing on a turn table with her hands fully extended horizontally brings her hands close together to say 'namaste'; what happens?

- ☞ From angular momentum conservation principle, we

have, $I_1\omega_1 = I_2\omega_2$. When the small girl brings her hands close together to say 'namaste', the moment of inertia of the system decreases. To keep $I\omega = \text{constant}$ in the absence of external torque, angular velocity ' ω ' of the system increases. So, turn table rotates faster.

6. Moment of inertia is also called the rotational inertia. Why?

- ☞ We know, the moment of inertia plays the same role in rotational motion as mass plays in translational (linear) motion. Greater the value of moment of inertia of a body, higher is the tendency to continue in the state of rotational motion. Hence, the moment of inertia is also called the rotational inertia.

7. The cap of the bottle can be easily opened with the help of two fingers than with one finger, why? [HSEB 2069]

- ☞ When the cap of the bottle is opened with the help of two fingers, then a couple is formed. But, when it is opened with the help of a single finger, then a torque is formed. We know, the torque due to a couple is double of the torque due to a single force. Hence, the cap is opened easily with the help of two fingers.

8. Explain why spokes are fitted in the cycle wheel. [HSEB 2056]

- ☞ When spokes are fitted in the cycle wheel, then the most of the mass of the cycle is concentrated on its rim. This increases the moment of inertia of the cycle. Such a large value of moment of inertia results the uniform motion on the cycle reducing jerks.

9. If the earth is struck by meteorites, the earth will slow down slightly, why? [HSEB 2053]

- ☞ Our earth is rotating about its polar axis by following the principle of conservation of angular momentum, i.e. $I\omega = \text{constant}$
If the earth is struck by meteorites, then its effective

mass increases and hence its moment of inertia also increases. Consequently, the angular velocity (ω) is decreased. So, the earth will slow down slightly.

10. Can a single force applied to a body change both its translational and rotational motion? Explain.

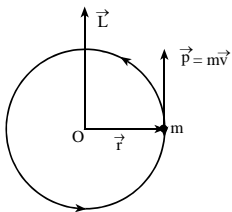
[HSEB 2068, S]

- Yes, when a body is rolling without slipping on a horizontal plane surface under the application of a force, then it rotates about the horizontal axis through its C.M. and it also undergoes displacement in the forward direction. Here, the body rotates about an horizontal axis and at the same time the C.M. of the body covers linear displacement. Hence, the body can have the change in both translational (linear) and rotational motion when a single force is applied.

11. Does the angular momentum of a body moving in a circular path change? Give explanation to your answer.

[NEB 2074]

- The angular momentum \vec{L} of a body having mass m moving in a circle of radius vector \vec{r} and having linear momentum \vec{P} is, $\vec{L} = \vec{r} \times \vec{P} = rp \sin \theta \hat{n}$



Where θ is angle between \vec{r} and \vec{P} and \hat{n} is the unit vector perpendicular to both \vec{r} and \vec{P} i.e. along the axis of rotation.

In uniform circular motion, $\theta = 90^\circ$

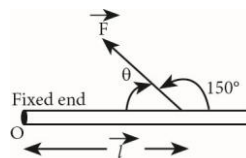
$$\therefore \vec{L} = rp \sin 90^\circ \hat{n} = rp \hat{n}$$

Here r and P are also constants. Hence, there is maximum rotational effect on the body and the angular momentum (L) remains constants.

12. One end of a solid rod of length L (in m) is fixed at one end. Can the magnitude and direction of torque be estimated if a force F (in N) act at a point l (in m) from the fixed end making an angle 150° with the horizontal? Explain.

[NEB 2074]

- Since, torque $\vec{\tau}$ is,



$$\vec{\tau} = \vec{r} \times \vec{F} = rF \sin \theta \hat{n}$$

And, $\tau = rF \sin \theta$ (in magnitude).

$$\text{or, } \tau = lF \sin (180 - 150^\circ)$$

$$\text{or, } \tau = lF \sin 30^\circ = lF \times \frac{1}{2}$$

$$\therefore \tau = \frac{lF}{2}$$

Here, the magnitude of torque is $\frac{lF}{2}$ which is

perpendicular to the plane of \vec{l} and \vec{F} i.e. if \vec{l} and \vec{F} are in the plane of page, then $\vec{\tau}$ is out of page. The direction of rotation is clockwise.

13. A solid sphere and a hollow cylinder of same mass and same size are rolling down on an inclined plane from rest. Which one reaches the ground first? Why?

[HSEB 2069 old, B]

- The acceleration attained by a body rolling down on an inclined plane is $a = \frac{g \sin \theta}{1 + \frac{I}{mr^2}}$

$$\text{For a hollow cylinder, } I = mr^2, a_c = \frac{g \sin \theta}{1 + \frac{mr^2}{mr^2}} = \frac{g \sin \theta}{2}$$

$$\text{For a solid sphere, } I = \frac{2}{5} mr^2, a_s = \frac{g \sin \theta}{1 + \frac{2}{5}} = \frac{5g \sin \theta}{7}$$

Here, $a_s > a_c$. So, the solid sphere reaches the ground first than the hollow cylinder.

14. A ballet dancer can increase or decrease her spinning rate by using the principle of conservation of angular momentum, how?

[HSEB 2068 old]

- From principle of conservation of angular momentum, $I_1 \omega_1 = I_2 \omega_2$

$$\text{Also, } I = mr^2$$

- When she lowers her hands, then her moment of inertia ($I_2 = mr^2$) decreases and hence her angular velocity (ω_2) increases. As ω_2 increases, the spinning rate is also increased.
- When she stretches her hands outward, then her moment of inertia ($I_2 = mr^2$) increases and hence ω_2 decreases. When ω_2 decreases, the spinning rate is also decreased.

15. Why is it easier to hold down a 10 kg body in your hand at your side than to hold it with your arm extended horizontally? [HSEB 2050]

✎ If a 10 kg body is held in our hand with our arm extended horizontally, the force i.e. weight of 10 kg mass and the distance of force from the axis of rotation are perpendicular. Hence, a maximum torque is produced. But, if the body is held at our side, then the lines of action of force (i.e. weight) pass through the rotation axis. So, a very small turning effect about the shoulder joint (rotation axis) is produced. Hence, it is easier to hold down a 10 kg mass in our hand at our side than to hold it with our arm extended horizontally.

16. If the ice on the polar caps of the earth melts, how will it affect the duration of the day? Explain. [HSEB 2073C]

✎ Since, no external torque is acting on the earth, its total angular momentum remains conserved, i.e.

$I\omega = \text{constant}$. Also, $I = mr^2$. When ice on the polar caps of the earth melts, then there is redistribution of its mass i.e. mass shifts towards the equatorial region, i.e. r increases from the polar axis of rotation of the earth. So, I increases and hence angular velocity (ω) of earth should decrease. When ω decreases, then the period (T) of rotation of earth increases. Hence, duration of the day increases.

17. Why is the most of the mass of a flywheel concentrated at its rim?

✎ The main purpose of a flywheel is to maintain uniform motion in spite of the changes in the accelerating or decelerating torques. For this, the moment of inertia (I) of the flywheel should be large. When the mass is concentrated at its rim, then the moment of inertia of the flywheel increases as given by the relation $I = \Sigma mr^2$. When such a wheel gains or loses some K.E. of rotation ($\text{K.E.} = \frac{1}{2} I\omega^2$) then the large value of ' I ' causes the small change in angular velocity of rotation ' ω '. This helps to maintain uniform rotational motion of the flywheel.

18. How is a swimmer jumping from a height in a swimming pool able to increase the no. of loops made in the air?

✎ From angular momentum conservation principle, $I\omega = \text{constant}$. To increase the no. of loops in air, ' ω ' should increase. To increase ω , moment of inertia (I) has to be decreased. This can be done by decreasing ' r ' of the body. Since, $I = \Sigma mr^2$, the swimmer has to curl his/her body in order to achieve more no. of loops in air.

19. A circular platform is rotating with a uniform angular velocity. What will be the change in motion of the platform if a person sits near the edge and then starts moving towards the center of the platform?

✎ From angular momentum conservation principle, $I\omega = \text{constant}$. Here, total moment of inertia of the system

is, $I_{\text{total}} = I_{\text{platform}} + mr^2$. If a person sits near the edge of the platform, then ' r ' and hence mr^2 will be more. This makes total moment of inertia I_{total} more. This makes angular velocity ' ω ' less and hence the circular platform rotates at a slower rate. When the person starts moving towards the center of the platform, then r and hence mr^2 decreases. It reduces I_{total} and increases ' ω '. So, the platform starts rotating at the faster rate.

20. A helicopter has two propellers, why?

✎ In order that the helicopter rise and fly, it has to have just one propeller which is able to throw air downwards. In doing so, the helicopter would turn in a certain direction. From angular momentum conservation principle, $I_1\omega_1 = I_2\omega_2$, the helicopter would then rotate in opposite direction, which is not practical. To stop this reverse rotation, a second propeller is used at the tail which pushes the air in opposite direction and hence stops the rotation. This second propeller at the tail makes the helicopter stable and safe.

21. Explain how a cat is able to land on its feet when thrown in to air?

✎ The cat uses the law of conservation of angular momentum, i.e. $I\omega = \text{constant}$. When the cat is thrown in air, it stretches its body together with the tail and feet so that the moment of inertia (I) increases. When I increases, then the angular velocity ω will be small and hence the cat can land on safely on its feet.

22. If the earth contracts to half its radius, what would be the length of one day?

✎ Earth is assumed to be a perfect sphere and its moment of inertia I is, $I = \frac{2}{5}MR^2$

If the earth contracts to half of its radius, then its moment of inertia becomes, $I' = \frac{2}{5}M\left(\frac{R}{2}\right)^2 = \frac{2}{5}\frac{MR^2}{4}$

From angular momentum conservation principle, $I'\omega' = I\omega$

$$\text{or, } \frac{2}{5}\frac{MR^2}{4} \cdot \omega' = \frac{2}{5}MR^2 \cdot \omega$$

$$\text{or, } \omega' = 4\omega$$

$$\text{or, } \frac{2\pi}{T'} = 4\left(\frac{2\pi}{T}\right)$$

$$\text{or, } T' = \frac{T}{4} = \frac{24}{4} = 6 \text{ hrs.}$$

Thus, the length of one day would be 6 hrs if earth contracts to half radius.

23. When calculating the moment of inertia of an object, can we treat all its mass as if it were concentrated at the center of mass of the object? Justify your answer.

✎ No, we can't consider the total mass to be concentrated at the centre of mass in rotational motion. This is

because the moment of inertia of a body is equal to the sum of moments of inertia of all the particles of the body and its value depends on the locations of the axis of rotations, total mass and the distribution of mass w.r.t. axis of rotation.

24. The work done by a force is the product of force and distance. The torque due to the force is the product of force and distance. Does this mean that torque and work are equivalent? Explain. [HSEB 2071 S]

✎ No, they are not equivalent. We have, work done W is,
 $W = \vec{F} \cdot \vec{S} = FS\cos\theta$, where S is displacement and θ is angle between force and displacement. Work is a scalar quantity. But, torque τ is, $\vec{\tau} = \vec{F} \times \vec{d} = Fd\sin\theta \hat{n}$, where d is the distance between the line of action of force and axis of rotation. Torque τ is a vector quantity. Unit of work is Nm or Joule but unit of torque is Nm but not Joule.

25. A flywheel rotates with constant angular velocity. Does a point on its rim have a tangential acceleration? A radial acceleration? Are these acceleration constant in magnitude? In direction? In each case, give the reasoning behind your answer. [HSEB 2070]

✎ Here, angular velocity, $\omega = \text{constant}$. So, tangential acceleration, $a_T = r\alpha = r\frac{d\omega}{dt} = 0$ and radial acceleration, $a = r\omega^2 = \text{constant}$. Thus, the tangential component of acceleration is zero and radial component of acceleration has a constant magnitude. The direction of radial acceleration changes at each point and it is directed towards the radius through each point of the circular path. The radial acceleration changes the direction of velocity and tangential acceleration changes the magnitude of velocity in rotational motion.



Worked Out Examples

1. A ballet dancer spins with 2.4 rev/sec with her arms outstretched when the moment of inertia about the axis of rotation is I_1 . With her arms folded, the moment of inertia about the same axis becomes $0.6I_1$. Calculate, the new rate of spin. [HSEB 2069A]

Solution:

Given, Initial frequency (f_1) = 2.4 rev/sec
 Initial moment of inertia (I_1) = I
 Final moment of inertia (I_2) = $0.6I$
 Final frequency (f_2) = ?

From angular momentum conservations principle,

$$I_2\omega_2 = I_1\omega_1$$

$$\text{or, } I_2 \times 2\pi f_2 = I_1 \times 2\pi f_1$$

$$\therefore f_2 = \frac{I_1 \times f_1}{I_2} = \frac{I \times 2.4}{0.6I} = 4 \text{ rev/sec.}$$

2. A constant torque 200 Nm turns a wheel about its centre. The moment of inertia about the axis is 100 kg m^2 . Find the kinetic energy gained after 20 revolutions when it starts from rest. [HSEB 2069 A, 2055]

Solution:

Given, Constant torque (τ) = 200 Nm
 Moment of inertia (I) = 100 kg m^2
 Number of revolution (n) = 20
 Kinetic energy gained (K.E) = ?
 Since, $\tau = I\alpha$

$$\text{or, } \alpha = \frac{\tau}{I} = \frac{200}{100} = 2 \text{ rad s}^{-2}$$

If θ be the angular displacement in 20 revolutions, then,

$$\theta = 2\pi n = 2\pi \times 20 = 40\pi \text{ rad}$$

Here, $\omega_0 = 0$ (initial angular velocity)

ω = angular velocity after 20 revolutions.

$$\text{Using } \omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\text{or, } \omega^2 = 0^2 + 2 \times 2 \times 40\pi = 160\pi$$

$$\therefore \omega^2 = 160\pi.$$

So, kinetic energy gained,

$$\begin{aligned} \text{K.E.} &= \frac{1}{2} I\omega^2 = \frac{1}{2} \times 100 \times 160\pi \\ &= 8000\pi \text{ Joules} = 25132.82 \text{ Joules.} \end{aligned}$$

3. An electric fan is turned off, and its angular velocity decreases uniformly from 500 rev/min to 200 rev/min in 4.00s. (a) Find the angular acceleration and the number of revolutions made by the motor in 4.00 s. interval. (b) How many more seconds are required for the fan to come to rest if the angular acceleration remains constant? [HSEB 2067]

Solution:

Given, Initial frequency (f_0) = 500 rev/min
 $= \frac{500}{60} \text{ rev/sec} = \frac{25}{3} \text{ rev/sec.}$

Final frequency (f) = 200 rev/min

$$= \frac{200}{60} \text{ rev/sec} = \frac{10}{3} \text{ rev/sec.}$$

Time (t) = 4 sec

$$\begin{aligned}\text{Initial angular velocity } (\omega_0) &= 2\pi f_0 = 2\pi \times \frac{25}{3} \\ &= 52.3 \text{ rad/sec.}\end{aligned}$$

$$\text{Final angular velocity } (\omega) = 2\pi f = 2\pi \times \frac{10}{3} = 20.9 \text{ rad/sec.}$$

a. Angular acceleration (α) = ?

Number of revolutions made in 4 sec, (n) = ?

We have,

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{20.9 - 52.3}{4} = -7.85 \text{ rad/s}^2$$

If θ be the angular displacement, then ,

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = 52.3 \times 4 - \frac{1}{2} \times 7.85 \times 4^2 = 146.4 \text{ rad}$$

Again, $\theta = 2\pi n$

$$\text{or, } n = \frac{\theta}{2\pi} = \frac{146.4}{2 \times 3.14} = 23.3 \text{ revolutions}$$

b. When the fan comes at rest, then,

$$\omega' = 0, \omega_0 = 20.9 \text{ rad/sec.}$$

$t' = ?$ (time taken to come to rest)

$$\text{Since, } \omega' = \omega_0 + \alpha t'$$

$$\text{or, } t' = \frac{\omega' - \omega_0}{\alpha} = \frac{0 - 20.9}{-7.85} = 2.66 \text{ sec}$$

So, the fan takes 2.66 sec more to come to rest.

4. A ballet dancer spins about a vertical axis at 1 revolution per second with her arms stretched. With her arms folded, her moment of inertia about this axis decreases by 40%, calculate the new rate of revolution. [HSEB 2067 S]

Solution:

Given, Initial frequency (f_1) = 1 rev s^{-1}

Initial moment of inertial (I_1) = I (suppose)

Final moment of inertia (I_2) = $I - 40\%$ of $I = 0.6 I$;

Final frequency (f_2) = ?

From angular momentum conservation principle,

$$I_2 \omega_2 = I_1 \omega_1$$

$$\text{or, } I_2 \times 2\pi f_2 = I_1 \times 2\pi f_1$$

$$\text{or, } f_2 = \frac{I_1 \times f_1}{I_2} = \frac{I \times 1}{0.6I} = 1.67 \text{ rev } s^{-1}$$

Hence, the final frequency (rate of revolutions) is 1.67 rev s^{-1}

5. A disc of moment of inertia $5 \times 10^{-4} \text{ kgm}^2$ is rotating freely about the axis through its centre at 40 rpm. Calculate the new revolutions per minute if some wax of mass 0.02 kg is dropped gently on to the disc 0.08 m from the axis. [HSEB 2066]

Solution:

Given, Initial moment of inertia (I_1) = $5 \times 10^{-4} \text{ kgm}^2$

Initial frequency of the disc (f_1) = 40 rpm

Final frequency of the disc (f_2) = ?

Mass of the wax (m) = 0.02 kg

Distance of wax from axis (r) = 0.08 m

Final moment of inertia (I_2) = $I_1 + mr^2$

From angular momentum conservation principle,

$$I_2 \omega_2 = I_1 \omega_1$$

$$\text{or, } (I_1 + mr^2) 2\pi f_2 = I_1 (2\pi f_1)$$

$$\text{or, } f_2 = \frac{I_1 f_1}{I_1 + mr^2} = \frac{5 \times 10^{-4} \times 40}{5 \times 10^{-4} + 0.02 \times 0.08^2} = 32 \text{ rpm.}$$

Hence, new frequency (revolution per minute) of the disc is $f_2 = 32 \text{ rpm}$

6. A constant torque of 500 Nm turns a wheel about its centre. The moment of inertia about this axis is 20 kgm^2 . Find the angular velocity and kinetic energy gained in 2 seconds. [HSEB 2061, 2054, 2072 C, 2070]

Solution:

Constant torque (τ) = 500 Nm

Moment of inertia (I) = 20 kgm^2 .

Angular velocity gained (ω) = ?

K.E. gained (K.E.) = ?

Time taken (t) = 2 sec

Initial angular velocity (ω_0) = 0

We have, $\tau = I\alpha$

$$\text{or, } \tau = I \frac{(\omega - \omega_0)}{t}$$

$$\text{or, } 500 = 20 \frac{(\omega - 0)}{2} = 10\omega$$

$$\therefore \omega = 50 \text{ rad/sec}$$

$$\text{Again, K.E.} = \frac{1}{2} I \omega^2 = \frac{1}{2} \times 20 \times 50^2 = 25000 \text{ Joules.}$$

Hence, angular velocity and K.E. gained in 2 sec are 50 rad/sec and 25,000 Joules respectively.

7. Speed of a body spinning about an axis increases from rest to 100 rev.min^{-1} in 5 sec, if a constant torque of 20 Nm is applied. The external torque is then removed and the body comes to rest in 100 sec due to friction. Calculate the frictional torque. [HSEB 2052]

Solution:

First case: When a constant torque is applied,

Torque (τ) = 20 Nm

Initial frequency (f_0) = 0

Final frequency (f) = 100 rev. min^{-1}

$$= \frac{100}{60} \text{ rev/s} = \frac{5}{3} \text{ rev/s}^{-1}$$

Time (t) = 5 sec;

Here, $\omega_0 = 2\pi f_0 = 0$

$$\omega = 2\pi f = 2\pi \times \frac{5}{3} = \frac{10\pi}{3} \text{ rad. s}^{-1}$$

We know, $\tau = I\alpha$

$$\text{or, } \tau = I \frac{(\omega - \omega_0)}{t}$$

$$\text{or, } 20 = I \frac{\left(\frac{10\pi}{3} - 0\right)}{5} = I \times \frac{10\pi}{3} \times \frac{1}{5} = I \times \frac{2\pi}{3}$$

$$\therefore I = \frac{30}{\pi} \text{ kg m}^2$$

Second case: When the torque is removed,

$$t' = 100 \text{ sec}$$

$$\omega' = 0$$

$$\omega_0 = \frac{10\pi}{3} \text{ rad. s}^{-1}$$

$$\text{Frictional torque } (\tau') = ?$$

Therefore,

$$\tau' = I\alpha' = I \frac{(\omega' - \omega_0)}{t'} = \frac{30}{\pi} \frac{(0 - 10\pi/3)}{100}$$

$$\text{or, } \tau' = -\frac{30}{\pi} \times \frac{10\pi}{3} \times \frac{1}{100} = -1 \text{ Nm.}$$

Hence, the magnitude of the frictional torque is 1 Nm.

8. What is the power output in horsepower of an electric motor turning at 4800 rev/min and developing a torque of 4.30 Nm?

Solution:

$$\text{Power output } (P) = ?$$

$$\text{Angular frequency } (f) = 4800 \text{ rev/min}$$

$$= \frac{4800}{60} \text{ rev/s} = 80 \text{ rev/s}$$

$$\text{Angular velocity } (\omega) = 2\pi f = 2\pi \times 80 = 503 \text{ rad s}^{-1}$$

$$\text{Torque } (\tau) = 4.30 \text{ Nm}$$

$$\text{Since, } P = \tau\omega = 4.30 \times 503$$

$$= 2163 \text{ watts} = \frac{2163}{746} \text{ HP} = 2.9 \text{ HP}$$

9. The flywheel of a gasoline engine is required to give up 500 J of kinetic energy while its angular velocity decreases from 650 rev/min to 520 rev/min. What moment of inertia is required?

Solution:

$$\text{Decrease in K.E.} = 500 \text{ J}$$

$$\text{Initial frequency } (f_0) = 650 \text{ rev/min}$$

$$= \frac{650}{60} \text{ rev/s} = 10.83 \text{ rev/s}$$

$$\text{Final frequency } (f) = 520 \text{ rev/min}$$

$$= \frac{520}{60} \text{ rev/s} = 8.67 \text{ rev/s}$$

$$\text{Moment of inertia } (I) = ?$$

$$\text{Initial angular velocity } (\omega_0) = 2\pi f_0 = 2 \times 3.14 \times 10.83 = 68.01 \text{ rad s}^{-1}$$

$$\text{Final angular velocity } (\omega) = 2\pi f = 2 \times 3.14 \times 8.67 = 54.45 \text{ rad s}^{-1}$$

$$\text{Since, decrease in K.E.} = \frac{1}{2} I \omega_0^2 - \frac{1}{2} I \omega^2$$

$$\text{or, } 500 = \frac{1}{2} I (\omega_0^2 - \omega^2)$$

$$\therefore I = \frac{500 \times 2}{\omega_0^2 - \omega^2} = \frac{1000}{(68.01)^2 - (54.45)^2} = 0.602 \text{ kg m}^2.$$

10. The main rotor of a helicopter is turning in a horizontal plane at 90.0 rev/min, the distance from the centre of the rotor shaft to each blade tip is 5.00 m. Calculate the speed through the air of the blade tip if (a) the helicopter is sitting on the ground; (b) the helicopter is rising vertically at 4.00 m/s.

Solution:

$$\text{Angular frequency } (f) = 90 \text{ rev/min} = \frac{90}{60} \text{ rev/s} = \frac{3}{2} \text{ rev/s.}$$

$$\text{Angular velocity } (\omega) = 2\pi f = 2 \times 3.14 \times \frac{3}{2} = 9.42 \text{ rad/s}$$

$$\text{Radius of the path of blade } (r) = 5 \text{ m}$$

$$\text{a. Speed of the blade at ground } (v) = ?$$

$$\text{b. Speed of blade rising } (v_T) = ?$$

$$\text{Rising velocity of the helicopter } (v_1) = 4 \text{ ms}^{-1}$$

$$\text{Since, } v = r\omega = 5 \times 9.42 = 47.10 \text{ ms}^{-1}$$

$$\text{Again, } v_T = \sqrt{v^2 + v_1^2} = \sqrt{(47.1)^2 + 4^2} = 47.3 \text{ ms}^{-1}$$

11. When drilling a 12.7 mm diameter hole in wood the required speed is 1250 rev/min. Determine its linear speed and radial acceleration.

Solution:

$$\text{Diameter of hole } (d) = 12.7 \text{ mm} = 12.7 \times 10^{-3} \text{ m}$$

$$\text{Radius of hole } (r) = \frac{d}{2} = 6.35 \times 10^{-3} \text{ m}$$

$$\text{Angular frequency } (f) = 1250 \text{ rev/min}$$

$$= \frac{1250}{60} \text{ rev/s} = \frac{125}{6} \text{ rev/s}$$

$$\text{Angular speed } (\omega) = 2\pi f = 2 \times 3.14 \times \frac{125}{6} = 131 \text{ rads}^{-1}$$

$$\text{Linear speed } (v) = ?$$

$$\text{Radial acceleration } (a_{\text{rad}}) = ?$$

$$\text{Since, } v = r\omega = 6.35 \times 10^{-3} \times 131 = 0.832 \text{ ms}^{-1}$$

$$\text{Again, } a_{\text{rad}} = r\omega^2 = (6.35 \times 10^{-3}) \times (131)^2 = 109 \text{ ms}^{-2}$$

12. The earth, which is not a uniform sphere, has a moment of inertia of 0.3308 MR^2 about an axis through its north and south poles. It takes the earth 86,164 s to spin once about this axis. Calculate (a) the earth's kinetic energy due to its rotation about this axis and (b) the earth's kinetic energy due to its orbital motion around the sun. (mass of earth = $5.97 \times 10^{24} \text{ kg}$, Radius of the earth = $6.38 \times 10^6 \text{ m}$, Radius of orbit = $1.50 \times 10^{11} \text{ m}$, orbital period = 365.3 days).

Solution:

$$\text{Moment of inertia of earth } (I) = 0.3308 \text{ MR}^2$$

Time period of rotation (T_r) = 86164 sec
 Orbital period (T_0) = $365.3 \times 24 \times 60 \times 60$ secs
 $= 31.56 \times 10^6$ secs.
 Rotational Kinetic energy ($K.E._r$) = ?
 Orbital kinetic energy ($K.E._0$) = ?
 Mass of the earth (M) = 5.97×10^{24} kg;
 Radius of the earth (R) = 6.38×10^6 m
 Radius of orbit of earth around the sun (R_0) = 1.50×10^{11} m
 Since,

$$K.E._r = \frac{1}{2} I \omega^2 = \frac{1}{2} 0.3308 M R^2 \left(\frac{2\pi}{T_r} \right)^2$$

$$= \frac{1}{2} \times 0.3308 \times 5.97 \times 10^{24} \times (6.38 \times 10^6)^2 \times \left(\frac{2 \times 3.14}{86144} \right)^2$$

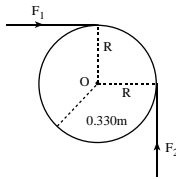
$$\therefore K.E._r = 2.14 \times 10^{29} \text{ J}$$

$$\text{And, } K.E._0 = \frac{1}{2} M V^2 = \frac{1}{2} M (R_0 \omega_0)^2 = \frac{1}{2} M R_0^2 \left(\frac{2\pi}{T_0} \right)^2$$

$$\text{or, } K.E._0 = \frac{1}{2} \times 5.97 \times 10^{24} \times (1.50 \times 10^{11})^2 \times \left(\frac{2 \times 3.14}{31.56 \times 10^6} \right)^2$$

$$\therefore K.E._0 = 2.66 \times 10^{33} \text{ J.}$$

- 13.** Forces $F_1 = 7.50 \text{ N}$ and $F_2 = 5.30 \text{ N}$ are applied tangentially to a wheel with radius 0.330 m , as shown in figure. What is the net torque on the wheel due to these two forces for an axis perpendicular to the wheel and passing through its center?



Solution:

$F_1 = 7.50 \text{ N}$, $F_2 = 5.30 \text{ N}$
 Radius of the wheel (R) = 0.330 m
 Net torque on the wheel (τ) = ?
 Since,
 $\tau = \tau_1 + \tau_2 = -F_1 R + F_2 R$
 $= (-F_1 + F_2) R = (-7.50 + 5.30) \times 0.330$
 $\therefore \tau = -0.726 \text{ Nm}$

- 14.** Emilie's potter's wheel rotates with a constant 2.25 rad/s^2 angular acceleration. After 4.00 s the wheel has rotated through an angle of 60.0 rad . What was the angular velocity of the wheel at the beginning of the 4.00 s interval?

Solution:

Constant angular acceleration (α) = 2.25 rad s^{-2}
 Time (t) = 4 sec

Angular displacement (θ) = 60 rad
 Initial angular velocity (ω_0) = ?

$$\text{Using, } \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\text{or, } 60 = \omega_0 \times 4 + \frac{1}{2} \times 2.25 \times 4^2$$

$$\therefore \omega_0 = 10.5 \text{ rad s}^{-1}$$

- 15.** A computer disk drive is turned on starting from the rest and has angular acceleration, (a) how long did it take to make the first complete rotation, and (b) what is the angular acceleration? Given that the disk took 0.750 sec for the drive to make its second complete revolution. [HSEB 2070 S]

Solution:

Given, Initial angular velocity (ω_0) = 0
 Time for 1st complete rotation (t_1) = ?
 Time for 2nd complete rotation = 0.75 sec .
 Angular acceleration (α) = ?
 Time for two complete rotations (t_2) = ($t_1 + 0.750$) secs.
 For 1st rotation, we have

$$\theta_1 = \omega_0 t_1 + \frac{1}{2} \alpha t_1^2$$

$$\text{or, } 2\pi = 0 + \frac{1}{2} \alpha t_1^2$$

$$\therefore 4\pi = \alpha t_1^2 \dots (1)$$

For two rotations, we have

$$\theta_2 = \omega_0 t_2 + \frac{1}{2} \alpha t_2^2$$

$$\text{or, } 4\pi = 0 + \frac{1}{2} \alpha (t_1 + 0.75)^2$$

$$\therefore 4\pi = \frac{1}{2} \alpha [t_1^2 + 2 \times 0.75 \times t_1 + (0.75)^2] \dots (2)$$

From equations (1) and (2), we get

$$\frac{1}{2} \alpha [t_1^2 + 1.50 t_1 + 0.5625] = \alpha t_1^2$$

$$\text{or, } t_1^2 + 1.50 t_1 + 0.5625 = 2 t_1^2$$

$$\therefore t_1^2 - 1.50 t_1 - 0.5625 = 0 \dots (3)$$

Solving equation (3), we get, $t_1 = 1.81 \text{ sec}$ and
 $t_1 = -0.31 \text{ sec}$ (Impossible)

$$\therefore \text{Time for 1st rotation } (t_1) = 1.81 \text{ sec}$$

Again, using value of t_1 in equation (1), we get,

$$4\pi = \alpha (1.81)^2$$

$$\therefore \alpha = \frac{4\pi}{3.2761} = 3.84 \text{ rad/sec}^2$$



Additional Numerical Examples

1. A flywheel is spinning at 500 rpm when a power fails. The power is off for 30.0 s and during this time the flywheel makes 200 complete revolutions. (a) What is the angular speed after 30 s? (b) What is the time taken to stop the wheel and how many revolutions are made during this time?

Solution:

$$\text{Initial frequency } (f_0) = 500 \text{ rpm} = \frac{500}{60} \text{ rev/s} = \frac{25}{3} \text{ rev/s}$$

$$\text{Time } (t) = 30.0 \text{ sec.}$$

$$\text{Number of complete revolutions made } (n) = 200$$

- a. $\omega = ?$ after 30 secs

$$\omega_0 = 2\pi f_0 = 2 \times 3.14 \times \frac{25}{3} = 52.4 \text{ rad s}^{-1}$$

$$\text{Angular displacement,}$$

$$\theta = 2\pi n = 2 \times 3.14 \times 200 = 1256.6 \text{ rad}$$

$$\text{Now, } \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\text{or, } 1256.6 = 52.4 \times 30 + \frac{1}{2} \alpha \times 30^2$$

$$\therefore \alpha = -0.701 \text{ rad s}^{-2}$$

$$\text{Again, we have}$$

$$\omega = \omega_0 + \alpha t = 52.4 - 0.701 \times 30$$

$$\therefore \omega = 31.4 \text{ rad s}^{-1}$$

$$\text{And, } f = \frac{\omega}{2\pi} = \frac{31.4}{2\pi} \text{ rev s}^{-1}$$

$$\therefore f = \frac{31.4}{2\pi} \times 60 \text{ rpm} = 300 \text{ rpm.}$$

- b. Time for stop (t_1) = ?

$$\text{No. of revolutions } (n_1) = ?$$

$$\text{Using, } \omega_1 = \omega_0 + \alpha t_1$$

$$\text{or, } 0 = 52.4 - 0.701 \times t_1$$

$$\therefore t_1 = 74.8 \text{ sec}$$

$$\text{Again, we have,}$$

$$\omega_1^2 = \omega_0^2 + 2\alpha\theta_1$$

$$\text{or, } 0 = (52.4)^2 - 2 \times 0.701 \times \theta_1$$

$$\therefore \theta_1 = 1958.4 \text{ rad}$$

$$\text{Since, } \theta_1 = 2\pi n_1$$

$$\text{or, } n_1 = \frac{\theta_1}{2\pi} = \frac{1958.4}{2 \times 3.14} = 312 \text{ revolutions.}$$

$$\therefore n_1 = 312 \text{ revolutions.}$$

2. The rotating blade of a blender turns with constant angular acceleration 1.50 rad/s^2 . (a) How much time does it take to reach an angular velocity of 36.0 rad/s , starting from rest? (b) Through how many revolutions does the blade turn in this time interval?

Solution:

$$\text{Constant angular acceleration } (\alpha) = 1.50 \text{ rad/s}^2$$

- a. Time required (t) = ?

$$\text{Final angular velocity } (\omega) = 36 \text{ rad/s}$$

$$\text{Initial angular velocity } (\omega_0) = 0$$

$$\text{Using, } \omega = \omega_0 + \alpha t$$

$$\text{or, } t = \frac{\omega - \omega_0}{\alpha} = \frac{36 - 0}{1.5}$$

$$\therefore t = 24.0 \text{ sec.}$$

- b. No. of revolutions made (n) = ?

$$\text{Using, } \omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\text{or, } (36)^2 = 0^2 + 2 \times 1.50 \times \theta$$

$$\therefore \theta = 432 \text{ rad}$$

$$\text{Now, } \theta = 2\pi n$$

$$\text{or, } n = \frac{\theta}{2\pi} = \frac{432}{2 \times 3.14}$$

$$\therefore n = 68.8 \text{ revolutions}$$

3. A safety device brings the blade of a power mower from an initial angular speed of ω_1 to rest in 1.00 revolution. At the same constant acceleration, how many revolutions would it take the blade to come to rest from an initial angular speed ω_3 that was three times as great, $\omega_3 = 3\omega_1$?

Solution:

Case I

$$\text{Initial angular speed} = \omega_1,$$

$$\text{number of revolution } n = 1 \text{ rev,}$$

$$\text{Final angular speed} = \omega_2 = 0$$

$$\text{Angular displacement } (\theta) = 2\pi n = 2\pi \times 1 = 2\pi \text{ rad}$$

$$\text{Let, } \alpha \text{ be the constant angular acceleration, then}$$

$$\omega_2^2 = \omega_1^2 + 2\alpha\theta$$

$$\text{or, } 0^2 = \omega_1^2 + 2\alpha \times 2\pi$$

$$\therefore \alpha = -\frac{\omega_1^2}{4\pi} \text{ rev/s}^2$$

Case II

$$\text{Now, initial angular speed } (\omega_3) = 3\omega_1$$

$$\text{Final angular speed } (\omega_4) = 0$$

$$\text{Number of revolution } (n_1) = ?$$

$$\text{Using, } \omega_4^2 = \omega_3^2 + 2\alpha\theta_1$$

$$\text{or, } 0^2 = (3\omega_1)^2 + 2\left(-\frac{\omega_1^2}{4\pi}\right) \cdot 2\pi n_1$$

$$\therefore n_1 = 9 \text{ revs.}$$

4. A flywheel with a radius of 0.300 m starts from rest and accelerates with a constant angular acceleration of 0.600 rad/s^2 . Compute the magnitude of the tangential acceleration, the radial acceleration and the resultant acceleration of a point on its rim (a) at the start; (b) after it has turned through 60.0°

Solution:

$$\text{Radius of the flywheel } (r) = 0.3 \text{ m}$$

$$\text{Initial angular velocity } (\omega_0) = 0$$

Constant angular acceleration (α) = 0.6 rad s^{-2}

a. At the start, $t = 0$,

Tangential acceleration (a_{tan}) = ?

Radial acceleration (a_{rad}) = ?

Resultant acceleration (a) = ?

Since, $a_{\text{tan}} = r\alpha$

or, $a_{\text{tan}} = 0.3 \times 0.6 = 0.18 \text{ ms}^{-2}$

Also, $a_{\text{rad}} = r\omega_0^2 = 0$

So, $a = \sqrt{a_{\text{tan}}^2 + a_{\text{rad}}^2} = \sqrt{(0.18)^2 + 0^2} = 0.18 \text{ ms}^{-2}$

b. Angular displacement (θ) = $60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3} \text{ rad}$

Let, ω be the angular velocity when the angular displacement is θ . Then, using

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\therefore \omega^2 = 2\alpha\theta$$

Now, $a_{\text{rad}} = r\omega^2 = r \times 2\alpha\theta = 0.3 \times (2 \times 0.6 \times \frac{\pi}{3}) = 0.377 \text{ ms}^{-2}$

Also, $a_{\text{tan}} = r\alpha = 0.3 \times 0.6 = 0.18 \text{ ms}^{-2}$

And, $a = \sqrt{a_{\text{tan}}^2 + a_{\text{rad}}^2} = \sqrt{(0.18)^2 + (0.377)^2} = 0.418 \text{ ms}^{-2}$

5. A centrifuge can produce a radial acceleration of $3000g$ at 5000 rev/min . Calculate the required radius of the centrifuge.

Solution:

Radial acceleration (a_{rad}) = $3000g \text{ ms}^{-2}$

Angular frequency (f) = 5000 rev/min
 $= \frac{5000}{60} \text{ rev/s} = \frac{250}{3} \text{ rev/s}$

Angular velocity (ω) = $2\pi f$
 $= 2 \times 3.14 \times \frac{250}{3} = 524 \text{ rads}^{-1}$

Radius (r) = ?

Since, $a_{\text{rad}} = r\omega^2$

$$\therefore 3000g = r \times (524)^2$$

$$\text{or, } r = \frac{3000 \times 9.8}{(524)^2} = 0.107 \text{ m} = 10.7 \text{ cm}$$

6. Energy is to be stored in a 70.0 kg flywheel in the shape of a uniform solid disk with radius $R = 1.20 \text{ m}$. To prevent structural failure of the flywheel, the maximum allowed radial acceleration of a point on its rim is 3500 m/s^2 . What is the maximum kinetic energy that can be stored in the flywheel?

Solution:

Mass of the flywheel (M) = 70 kg

Radius (r) = 1.20 m

Maximum radial acceleration (a_{rad}) = 3500 ms^{-2}

Maximum kinetic energy stored (K.E.) = ?

Now, $a_{\text{rad}} = r\omega^2$

$$\text{or, } 3500 = 1.20 \times \omega^2$$

$$\therefore \omega^2 = \frac{3500}{1.20} = 2917. \text{rad}^2 \cdot \text{s}^{-2}$$

$$\text{And, } I = \frac{1}{2} mr^2 = \frac{1}{2} \times 70 \times (1.2)^2 = 50.4 \text{ kgm}^2$$

$$\text{So, K.E.} = \frac{1}{2} I\omega^2 = \frac{1}{2} \times 50.4 \times 2917 = 7.35 \times 10^4 \text{ J}$$

7. Your car's speedometer converts the angular speed of the wheels to the linear speed of the car, assuming standard size tyres and no slipping on the pavement. If your car's standard tyres are 24 inches in diameter, at what rate (in rpm) are your wheels rotating when you are driving at a freeway speed of 60 mph ?

Solution:

Tyres diameter (d) = 24 inches ; $1 \text{ mile} = 1609 \text{ m}$.

Tyres radius (r) = $\frac{d}{2} = 12 \text{ inches} = 12 \times 2.54 \text{ cm}$

$$= 30.5 \text{ cm} = 0.305 \text{ m}$$

Linear speed (v) = 60 mph (miles per hour)

$$= \frac{60 \times 1609}{60 \times 60} = 26.81 \text{ ms}^{-1}$$

Angular frequency (f) = ?

We have $v = r\omega$

$$\text{or, } \omega = \frac{v}{r} = \frac{26.81}{0.305} = 87.92 \text{ rad s}^{-1}$$

Also, $\omega = 2\pi f$

$$\therefore f = \frac{\omega}{2\pi} = \frac{87.92}{2 \times 3.14} = 13.99 \text{ rev s}^{-1}$$

$$= 13.99 \times 60 \text{ rpm} = 840 \text{ rpm}$$

8. A flywheel having a moment of inertia of 16.0 kg.m^2 and runs at 300 rev/min . (a) Find the speed in rev/min to which the flywheel will be reduced when 4000 J of work is required. (b) What must the constant power supply to the flywheel (in watts) be to bring it back to its initial speed in a time of 5.00 s ?

Solution:

M.I. of flywheel (I) = 16.0 kg m^2

Initial frequency (f_0) = 300 rev/min

$$= \frac{300}{60} \text{ rev/s} = 5 \text{ rev/s}$$

Initial angular velocity (ω_0) = $2\pi f_0$

$$= 2 \times 3.14 \times 5 = 31.4 \text{ rad s}^{-1}$$

a Work required (ω) = 4000 J ;

The required value of angular velocity (ω) = ?

Time taken (t) = 5 sec

Since, Change in K.E. = work required

$$\text{or, } \frac{1}{2} I\omega_0^2 - \frac{1}{2} I\omega^2 = W$$

$$\text{or, } \frac{1}{2} I\omega^2 = \frac{1}{2} I\omega_0^2 - W$$

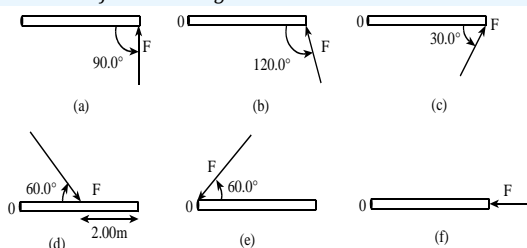
$$\therefore \omega = \sqrt{\omega_0^2 - \frac{2W}{I}} = \sqrt{(31.4)^2 - \frac{2 \times 4000}{16}} = 22.04 \text{ rad s}^{-1}$$

$$\text{and, } f = \frac{\omega}{2\pi} = \frac{22.04}{2 \times 3.14} = 3.50 \text{ rev/sec}$$

$$= 3.50 \times 60 \text{ rev/min} = 210.5 \text{ rev/min}$$

$$\text{b. Again, Power, } P = \frac{W}{t} = \frac{4000}{5} = 800 \text{ watts.}$$

9. Calculate the torque (magnitude and direction) about point O due to the force \vec{F} in each of the situations sketched in figures. In each case, the force \vec{F} and the rod lie in the plane of the page, the rod has length 4.00 m, and the force has magnitude $F = 10.0 \text{ N}$.



Solution:

Length of the rod (l) = $r = 4 \text{ m}$;

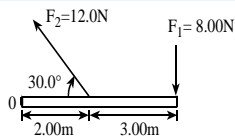
Applied force (F) = 10 N

$$\text{Since, } \vec{\tau} = \vec{r} \times \vec{F}$$

$$\therefore \tau = r F \sin \theta \text{ (in magnitude)}$$

- $\tau = r F \sin \theta = 4 \times 10 \times \sin 90^\circ = 40 \text{ Nm}$ (out of page)
- $\tau = r F \sin \theta = 4 \times 10 \times \sin 120^\circ = 34.64 \text{ Nm}$ (out of page)
- $\tau = r F \sin \theta = 4 \times 10 \times \sin 30^\circ = 20 \text{ Nm}$ (out of page)
- $\tau = r F \sin \theta = 2 \times (-10) \times \sin 60^\circ = -17.32 \text{ Nm}$ (into the page)
- $\tau = r F \sin \theta = 0 \times 10 \times \sin 60^\circ = 0$
- $\tau = r F \sin \theta = 4 \times 10 \times \sin 0^\circ = 0$

10. Calculate the net torque about point O for the two forces applied as in figure. The rod and both forces are in the plane of the page.



Solution:

$$F_1 = \text{N}$$

$$\theta_1 = 90^\circ$$

$$r_1 = (2 + 3) \text{ m} = 5 \text{ m}$$

$$F_2 = 12 \text{ N}$$

$$\theta_2 = 30^\circ$$

$$r_2 = 2 \text{ m}$$

$$\text{Net torque } (\tau) = ?$$

$$\text{Here, } \tau = \tau_1 + \tau_2$$

Since,

$$\tau_1 = r_1 F_1 \sin \theta_1 = 5 \times (-8) \times \sin 90^\circ = -40 \text{ Nm (Clockwise)}$$

$$\text{And, } \tau_2 = r_2 F_2 \sin \theta_2 = 2 \times 12 \times \sin 30^\circ = 12 \text{ Nm (anti-clockwise)}$$

$$\therefore \tau = \tau_1 + \tau_2 = -40 + 12 = -28 \text{ Nm (Clockwise)}$$

11. The flywheel of an engine has a moment of inertia 2.50 kgm^2 about its rotation axis. (a) What constant torque is required to bring up to an angular speed of 400 rev/min in 8.00 s , starting from rest? (b) What is its final kinetic energy?

Solution:

$$\text{M.I. of flywheel } (I) = 2.50 \text{ kgm}^2$$

$$\text{a. Constant torque } (\tau) = ?$$

$$\text{Initial angular velocity } (\omega_0) = 0$$

$$\text{Final frequency } (f) = 400 \text{ rev/min}$$

$$= \frac{400}{60} \text{ rev/sec}$$

$$= \frac{20}{3} \text{ rev/sec}$$

$$\text{Final angular velocity } (\omega) = 2\pi f$$

$$= 2 \times 3.14 \times \frac{20}{3}$$

$$= 41.89 \text{ rads}^{-1}$$

$$\text{Time } (t) = 8.00 \text{ sec.}$$

$$\therefore \tau = I\alpha$$

$$\text{or, } \tau = I \frac{(\omega - \omega_0)}{t} = 2.50 \times \frac{(41.89 - 0)}{8} = 13.09 \text{ Nm.}$$

$$\text{b. Final Kinetic energy (K.E.)} = ?$$

Since,

$$\text{K.E.} = \frac{1}{2} I \omega^2 = \frac{1}{2} \times 2.50 \times (41.89)^2$$

$$= 2193 \text{ J} = 2.193 \times 10^3 \text{ J}$$

12. A cord is wrapped around the rim of a wheel 0.250 m in radius, and a steady pull of 40.0 N is exerted on the cord. The wheel is mounted on frictionless bearings on a horizontal shaft through its centre. The moment of inertia of the wheel about this shaft is 5.00 kgm^2 . Compute the angular acceleration of the wheel.

Solution:

$$\text{Radius of the wheel } (R) = 0.250 \text{ m;}$$

$$\text{Force applied } (F) = 40 \text{ N}$$

$$\text{M.I. of the wheel } (I) = 5 \text{ kgm}^2;$$

$$\text{Angular acceleration of the wheel } (\alpha) = ?$$

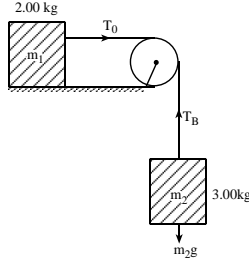
$$\text{Since, } \tau = I\alpha$$

$$\text{Also, } \tau = FR$$

$$\therefore I\alpha = FR$$

$$\text{or, } \alpha = \frac{FR}{I} = \frac{40 \times 0.250}{5} = 2 \text{ rad s}^{-2}$$

13. A 2.00 kg textbook rests on a frictionless, horizontal surface. A cord attached to the book passes over a pulley whose diameter is 0.150 m, to a hanging book with mass 3.00 kg. The system is released from rest, and the books are observed to move 1.20 m in 0.800 s. (a) What is the tension in each part of the cord? (b) What is the moment of inertia of the pulley about its rotation axis?



Solution:

Mass of textbook (m_1) = 2 kg

Diameter of pulley (d) = 0.150 m

$$\therefore \text{Radius of pulley } (r) = \frac{d}{2} = 0.075 \text{ m}$$

Mass of hanging book (m_2) = 3 kg

Distance moved by the system (s) = 1.20 m

Time (t) = 8.00 secs

- a. Tension on horizontal part (T_0) = ?

Tension on vertical part (T_B) = ?

Here, $T_0 = m_1 a \rightarrow$ (i)

And, $m_2 g - T_B = m_2 a \rightarrow$ (ii)

Also, using $s = ut + \frac{1}{2} a t^2$

$$\text{or, } s = \frac{1}{2} a t^2, [\text{since, } u = 0]$$

$$\text{or, } 1.20 = \frac{1}{2} \times a \times (0.8)^2$$

$$\therefore a = 3.75 \text{ ms}^{-2}$$

$$\therefore \text{From (i), } T_0 = 2 \times 3.75 = 7.5 \text{ N}$$

$$\text{And, from (ii), } T_B = m_2 (g - a) = 3(9.8 - 3.75)$$

$$\therefore T_B = 18.2 \text{ N}$$

- b. M.I. of the pulley (I) = ?

We have, $\tau = I\alpha$

Also, $\tau = rT_B - rT_0$

$$\therefore I\alpha = r(T_B - T_0)$$

$$\text{or, } I \left(\frac{a}{r} \right) = r(T_B - T_0) \quad [\because a = r\alpha]$$

$$\text{or, } I = \frac{r^2(T_B - T_0)}{a} = \frac{(0.075)^2 \times (18.2 - 7.5)}{3.75}$$

$$\therefore I = 0.0161 \text{ kgm}^2$$

14. A playground merry-go-round has radius 2.40 m and a moment of inertia 2100 kgm² about a vertical axis through its centre, and turns with negligible friction. (a) A child applies an 18.0 N force tangentially to the edge of the merry-go-round for 15.0 s. If the merry-go-round is initially at rest, what is its angular speed after this 15.0 s interval? (b) How much work did the child do on the merry-go-round? (c) What is the average power supplied by the child?

Solution:

Radius (r) = 2.40 m

Moment of inertia (I) = 2100 kgm²

- a. Force applied (F) = 18 N

Time (t) = 15 sec

Initial angular speed (ω_0) = 0

Final angular speed (ω) = ?

Since, $\tau = I\alpha$ Also, $\tau = rF$

$$\therefore I\alpha = rF$$

$$\text{or, } \alpha = \frac{rF}{I} = \frac{2.40 \times 18}{2100} = 0.0206 \text{ rads}^{-2}$$

Again,

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = 0 \times 15 + \frac{1}{2} \times 0.0206 \times (15)^2 = 2.32 \text{ rad}$$

Now, $\omega^2 = \omega_0^2 + 2\alpha\theta$

$$\text{or, } \omega^2 = 0 + 2 \times 0.0206 \times 2.32$$

$$\therefore \omega = 0.309 \text{ rad s}^{-1}$$

- b. Work done (W) = ?

- c. Average Power (P_{av}) = ?

$$\text{Since, } W = \tau\theta = rF\theta = 2.40 \times 18 \times 2.32 = 100.2 \text{ J}$$

$$\text{And, } P_{av} = \frac{W}{t} = \frac{100.2}{15} = 6.68 \text{ watts}$$

15. A diver comes off a board with arms straight up and legs straight down, giving her a moment of inertia about her rotation axis of 18 kgm². She then tucks into a small ball, decreasing this moment of inertia to 3.6 kgm². While tucked, she makes two complete revolutions in 1.0 s. If she hadn't tucked at all, how many revolutions would she have made in the 1.5 s from board to water?

Solution:

$$I_1 = 18 \text{ kgm}^2$$

$$I_2 = 3.6 \text{ kgm}^2$$

$$n_1 = ?$$

$$f_2 = 2 \text{ revolutions/sec}$$

$$\omega_1 = ?$$

$$\omega_2 = 2\pi f_2 = 2\pi \times 2 = 4\pi \text{ rads}^{-1}$$

$$t_1 = 1.5 \text{ sec}$$

$$t_2 = 1.0 \text{ sec}$$

Using angular momentum conservation principle,

$$I_1\omega_1 = I_2\omega_2$$

$$\text{or, } 18 \times \omega_1 = 3.6 \times 4\pi$$

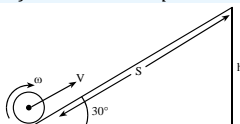
$$\therefore \omega_1 = 2.51 \text{ rad s}^{-1}$$

$$\text{Now, } \theta_1 = 2\pi n_1$$

$$\text{or, } \omega_1 t_1 = 2\pi n_1$$

$$\therefore n_1 = \frac{\omega_1 t_1}{2\pi} = \frac{2.51 \times 1.5}{2 \times 3.14} = 0.60 \text{ rev.}$$

16. A solid disk is rolling without slipping on a level surface at a constant speed of 2.50 m/s. If the disk rolls up a 30.0° ramp, how far along the ramp will it move before it stops?



Solution:

Speed of solid disk (v) = 2.50 ms⁻¹,

Angle of inclination (θ) = 30°

Distance travelled along the incline (s) = ?

Here, Total K.E. is converted in to P.E. That is,
Loss in K.E. = Gain in P.E.

$$\text{or, } \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\text{or, } \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 = mgh$$

$$\text{or, } v^2 + \frac{1}{2}r^2\omega^2 = 2gh$$

$$\text{or, } v^2 + \frac{1}{2}v^2 = 2gh$$

$$\text{or, } \frac{3v^2}{2} = 2gh$$

$$\therefore h = \frac{3v^2}{4g} = \frac{3 \times (2.50)^2}{4 \times 9.8} = 0.478 \text{ m}$$

$$\text{Now, } \sin \theta = \frac{h}{s}$$

$$\therefore s = \frac{h}{\sin \theta} = \frac{0.478}{\sin 30^\circ} = 0.956 \text{ m}$$

17. A disc of moment of inertia 10 kgm² about its centre rotates steadily about the centre with an angular velocity of 20 rad s⁻¹. Calculate (a) its rotational energy, (b) its angular momentum about the centre, (c) the number of revolutions per second of the disc.

Solution:

Moment of inertia of disc (I) = 10 kgm²

Angular velocity (ω) = 20 rad s⁻¹

a. Rotational Energy ($K.E_{\text{rot}}$) = ?

We have,

$$K.E_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 10 \times 20^2 = 2000 \text{ J.}$$

b. Angular momentum (L) = ?

We have,

$$L = I\omega = 10 \times 20 = 200 \text{ kgm}^2 \text{ s}^{-1}$$

c. No. of revolutions per sec i.e. frequently (f) = ?

Since, $\omega = 2\pi f$

$$\therefore f = \frac{\omega}{2\pi} = \frac{20}{2\pi} = \frac{10}{\pi} = 3.2 \text{ rev/sec}$$

18. A flywheel has a kinetic energy of 200 J. Calculate the number of revolutions it makes before coming to rest if a constant opposing couple of 5 Nm is applied to the flywheel. If the moment of inertia of the flywheel about its centre is 4 kgm². How long does it take to come to rest?

Solution:

Kinetic energy of flywheel (K.E) = 200 J

No. of revolutions (n) = ?

Constant opposing torque (τ) = -5 Nm

Moment of inertia (I) = 4 kgm²

Time taken (t) = ?

Final angular velocity (ω) = 0

$$\text{Since, } K.E. = \frac{1}{2}I\omega_0^2$$

$$\text{or, } 200 = \frac{1}{2} \times 4 \times \omega_0^2$$

$$\therefore \omega_0 = 10 \text{ rad s}^{-1}$$

Also, $\tau = I\alpha$

$$\text{or, } -5 = 4 \times \alpha$$

$$\therefore \alpha = -1.25 \text{ rad s}^{-2}$$

Now using,

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\text{or, } 0^2 = 10^2 + 2 \times (-1.25) \times \theta$$

$$\therefore \theta = 40 \text{ rad.}$$

$$\text{Again, } \theta = 2\pi n$$

$$\text{or, } n = \frac{\theta}{2\pi} = \frac{40}{2 \times 3.14} = 6.4 \text{ rev}$$

The time taken to come to rest is,

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 10}{-1.25} = 8 \text{ sec.}$$

19. A disc rolling along a horizontal plane has a moment of inertia 2.5 kgm² about its centre and a mass of 5 kg. The velocity along the plane is 2 ms⁻¹. If the radius of the disc is 1 m, find (i) the angular velocity, (ii) the total energy (rotational and translational) of the disc. [HSEB 2072 Set E]

Solution:

$I = 2.5 \text{ kgm}^2$,

mass (m) = 5 kg

velocity (v) = 2ms⁻¹

radius (r) = 1 m

a. Angular velocity (ω) = ?

We have

$$v = r\omega$$

$$\text{or, } \omega = \frac{v}{r} = \frac{2}{1} = 2 \text{ rad s}^{-1}$$

b. Total energy (E) = ?

We have,

$$E = K.E_{\text{rot}} + K.E_{\text{trans}} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 2.5 \times 2^2 + \frac{1}{2} \times 5 \times 2^2$$

$$\therefore E = 15 \text{ Joules}$$

20. A wheel of moment of inertia 20 kgm^2 about its axis is rotated from rest about its centre by a constant torque τ and the energy gained in 10 s is 360 J . Calculate (i) the angular velocity at the end of 10 s , (ii) τ , (iii) the number of revolutions made by the wheel before coming to rest if τ is removed at 10 s and a constant opposing torque of 4 N is then applied to the wheel.

Solution:

Moment of inertia (I) = 20 kgm^2

Initial angular velocity (ω_0) = 0

Constant torque = τ ;

Time (t) = 10 secs

Energy gained (E) = 360 J

a. $\omega = ?$ after 10 sec

$$\text{Since, } E = \frac{1}{2} I \omega^2$$

$$\text{or, } \omega = \sqrt{\frac{2E}{I}} = \sqrt{\frac{2 \times 360}{20}}$$

$$\therefore \omega = 6 \text{ rad s}^{-1}$$

b. $\tau = ?$

We have,

$$\tau = I \alpha = I \frac{(\omega - \omega_0)}{t} = 20 \times \frac{(6 - 0)}{10}$$

$$\therefore \tau = 12 \text{ Nm}$$

c. No. of revolutions (n) = ?

Constant opposing torque (τ) = -4 N

Final angular velocity (ω') = 0

Initial angular velocity (ω_0') = 6 rad s^{-1}

Now, $\tau = I \alpha'$

$$\text{or, } -4 = 20 \times \alpha'$$

$$\therefore \alpha' = -0.2 \text{ rad s}^{-2}$$

$$\text{Also, } \omega'^2 = \omega_0'^2 + 2 \alpha' \theta$$

$$\text{or, } 0^2 = 6^2 - 2 \times 0.2 \times \theta$$

$$\therefore \theta = \frac{36}{0.4} = 90 \text{ rad}$$

$$\text{Now, } \theta = 2\pi n$$

$$\text{or, } n = \frac{\theta}{2\pi} = \frac{90}{2 \times 3.14} = 14.3 \text{ rev}$$

21. A uniform rod of length 3 m is suspended at one end so that it can move about an axis perpendicular to its length. The moment of inertia about the end is 6 kgm^2 and the mass of the rod is 2 kg . If the rod is initially horizontal and then released, find the angular velocity of the rod when (i) it is inclined at 30° to the horizontal; (ii) reaches the vertical.

Solution:

Length of rod (l) = 3 m

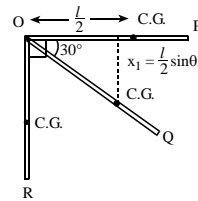
M.I. about end O (fixed end), $I = 6 \text{ kgm}^2$

mass of rod (m) = 2 kg

In the initial horizontal position OP, $\omega_0 = 0$

a. $\omega = ?$ when $\theta = 30^\circ$

Let, the C.G. at position OQ i.e. $\theta = 30^\circ$ is lowered down through x_1 as shown in figure.



Then, Loss in P.E. = Gain in K.E.

$$\text{or, } mgx_1 = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$$

$$\text{or, } mg \left(\frac{l}{2} \right) \sin \theta = \frac{1}{2} I \omega^2 + \frac{1}{2} m \left(\frac{l}{2} \omega \right)^2$$

$$\text{or, } mg \left(\frac{l}{2} \right) \sin \theta = \frac{1}{2} \omega^2 \left(I + \frac{m l^2}{4} \right) \quad \dots (1)$$

$$\therefore 2 \times 10 \times \left(\frac{3}{2} \right) \sin 30^\circ = \frac{1}{2} \omega^2 \left(6 + \frac{2 \times 3^2}{4} \right)$$

$$\therefore \omega = 1.69 \text{ rad s}^{-1}$$

b. $\omega = ?$ when $\theta = 90^\circ$

We have, from equation (1)

$$mg \left(\frac{l}{2} \right) \sin \theta = \frac{1}{2} \omega^2 \left(I + \frac{m l^2}{4} \right)$$

$$\text{or, } 2 \times 10 \times \left(\frac{3}{2} \right) \times \sin 90^\circ = \frac{1}{2} \omega^2 \times \left(6 + \frac{2 \times 3^2}{4} \right)$$

$$\therefore \omega = 2.39 \text{ rad s}^{-1}$$

22. A recording disc rotates steadily at 45 rev. min^{-1} on a table. When a small mass 0.02 kg is dropped gently on the disc at a distance of 0.04 m from its axis and sticks to the disc, the rate of revolution falls to 36 rev. min^{-1} . Calculate the moment of inertia of the disc about its centre.

Solution:

$$f_1 = 45 \text{ rev min}^{-1} = \frac{45}{60} \text{ rev s}^{-1} = \frac{3}{4} \text{ rev s}^{-1};$$

$$f_2 = 36 \text{ rev min}^{-1} = \frac{36}{60} \text{ rev s}^{-1} = \frac{3}{5} \text{ rev s}^{-1}$$

Moment of inertia of the disc (I_1) = ?

Mass dropped (m) = 0.02 kg ;

Distance from centre (r) = 0.04 m

New moment of inertia (I_2) = $I_1 + m r^2$

Now, using angular momentum conservation principle,

$$I_2 \omega_2 = I_1 \omega_1$$

$$\text{or, } (I_1 + m r^2) \times 2\pi f_2 = I_1 \times 2\pi f_1$$

$$\text{or } (I_1 + 0.02 \times 0.04^2) \times \frac{3}{5} = I_1 \times \frac{3}{4}$$

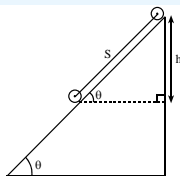
$$\text{or, } I_1 + 3.2 \times 10^{-5} = I_1 \times 5/4$$

$$\text{or, } I_1 \left(\frac{5}{4} - 1 \right) = 3.2 \times 10^{-5}$$

$$\text{or, } I_1 = 3.2 \times 10^{-5} \times 4$$

$$\therefore I_1 = 1.28 \times 10^{-4} \text{ kgm}^2$$

- 23.** A disc of moment of inertia 0.1 kgm^2 about its centre and radius 0.2 m is released from rest on a plane inclined at 30° to the horizontal. Calculate the angular velocity after it has rolled 2 m down the plane if its mass is 5 kg .



Solution:

$$\text{M.I. of disc (I)} = 0.1 \text{ kgm}^2$$

$$\text{Radius of disc (r)} = 0.2 \text{ m}$$

$$\omega_0 = 0, \theta = 30^\circ, \omega = ?$$

$$\text{Distance travelled (s)} = 2 \text{ m}$$

$$\text{Mass (m)} = 5 \text{ kg}$$

$$\text{Here, Loss in P.E.} = \text{Gain in K.E.}$$

$$mgh = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$$

$$\text{or, } mgs \sin \theta = \frac{1}{2} I \omega^2 + \frac{1}{2} m (r\omega)^2$$

$$\text{or, } mgs \sin \theta = \frac{1}{2} I \omega^2 + \frac{1}{2} m r^2 \omega^2$$

$$\text{or, } mgs \sin \theta = \frac{1}{2} (I + m r^2) \omega^2$$

$$\therefore \omega = \sqrt{\frac{2 mgs \sin \theta}{(I + m r^2)}} = \sqrt{\frac{2 \times 5 \times 10 \times 2 \times \sin 30^\circ}{(0.1 + 5 \times 0.2^2)}} = 18.3 \text{ rad s}^{-1}$$

- 24.** A flywheel with an axle 1.0 cm in diameter is mounted in frictionless bearings and set in motion by applying a steady tension of 2 N to a thin thread wound tightly round the axle. The moment of inertia of the system about its axis of rotation is $5.0 \times 10^{-4} \text{ kgm}^2$. Calculate (a) the angular acceleration of the flywheel when 1 m of thread has been pulled off the axle, (b) the constant retarding couple which must then be applied to bring the flywheel to rest in one complete turn, the tension in the thread have been completely removed.

Solution:

$$\text{Diameter of axle (d)} = 1 \text{ cm}$$

$$\text{Radius of the axle (r)} = \frac{d}{2} = 0.5 \text{ cm} = 5 \times 10^{-3} \text{ m}$$

$$\text{Force applied to the axle (F)} = T = 2 \text{ N}$$

$$\text{Moment of inertia of the system (I)} = 5 \times 10^{-4} \text{ kgm}^2$$

$$\text{Torque on the axle, } \tau = rF = rT = 5 \times 10^{-3} \times 2 = 0.01 \text{ Nm}$$

$$\text{a. Angular accelerations } (\alpha) = ?$$

$$\text{Since, } \tau = I\alpha$$

$$\text{or, } 0.01 = 5 \times 10^{-4} \times \alpha$$

$$\therefore \alpha = 20 \text{ rad s}^{-2}$$

$$\text{b. Constant retarding couple } (\tau') = ?$$

Let n be the no. of revolutions made by the axle when 1 m of the thread has been pulled off, then,

$$2\pi r \times n = 1$$

$$\text{or, } n = \frac{1}{2\pi r}$$

If θ be the angular displacement, then,

$$\theta = 2\pi n = 2\pi \times \frac{1}{2\pi r} = \frac{1}{r} = \frac{1}{5 \times 10^{-3}} = 200 \text{ rad}$$

Now, the angular velocity of the flywheel and the axle when 1 m of the thread has been pulled off is,

$$\omega^2 = \omega_0^2 + 2\alpha\theta = 0^2 + 2 \times 20 \times 200$$

$$\therefore \omega = 89.44 \text{ rad s}^{-1}$$

Now to bring the flywheel at rest in one complete turn,

$$\theta' = 1 \text{ turn} = 2\pi \text{ rad; } \omega'_0 = 89.44 \text{ rad s}^{-1}$$

$$\omega' = 0$$

$$\text{Now, } \omega'^2 = \omega'_0{}^2 + 2\alpha'\theta'$$

$$\text{or, } 0^2 = (89.44)^2 + 2 \times \alpha' \times 2\pi$$

$$\therefore \alpha' = -\frac{8000}{4\pi} \text{ rad s}^{-2}$$

$$\text{So, } \tau' = I\alpha' = 5 \times 10^{-4} \times \left(-\frac{8000}{4\pi} \right) = -32 \text{ Nm}$$

The negative sign shows the retarding couple

- 25.** A horizontal disc rotating freely about a vertical axis makes 100 r.p.m. A small piece of wax of mass 10 g falls vertically on to the disc and adheres to it at a distance of 9 cm from the axis. If the number of revolutions per minute is thereby reduced to 90 , calculate the moment of inertia of the disc.

Solution:

$$\text{Frequency of disc (f}_1\text{)} = 100 \text{ rpm} = \frac{100}{60} \text{ rev/s} = \frac{5}{3} \text{ rev/s}$$

$$\text{Mass of wax (m)} = 10 \text{ g} = 0.01 \text{ kg}$$

$$\text{Distance of wax from the axis (r)} = 9 \text{ cm} = 0.09 \text{ m}$$

$$\text{New frequency of disc with wax}$$

$$(f_2) = 90 \text{ rpm} = \frac{90}{60} \text{ rev/s} = \frac{3}{2} \text{ rev/s}$$

$$\text{M.I. of disc without wax (I}_1\text{)} = ?$$

$$\text{M.I. of disc with wax (I}_2\text{)} = I_1 + m r^2$$

From law of conservation of angular momentum,

$$I_2 \omega_2 = I_1 \omega_1$$

$$\text{or, } (I_1 + m r^2) \times 2\pi f_2 = I_1 \times 2\pi f_1$$

$$\text{or, } \frac{I_1 + m r^2}{I_1} = \frac{f_1}{f_2}$$

$$\text{or, } \frac{I_1 + 0.01 \times 0.09^2}{I_1} = \frac{5/3}{3/2} = \frac{10}{9}$$

$$\text{or, } 1 + \frac{0.01 \times 0.09^2}{I_1} = \frac{10}{9}$$

$$\text{or, } \frac{0.01 \times 0.09^2}{I_1} = \frac{10}{9} - 1 = \frac{1}{9}$$

$$\therefore I_1 = 9 \times 0.01 \times 0.09^2 = 7.3 \times 10^{-4} \text{ kgm}^2$$

- 26.** The atoms in the oxygen molecule O_2 may be considered to be point masses separated by a distance of $1.2 \times 10^{-10} \text{ m}$. The molecular speed of an oxygen molecule at s.t.p. is 460 ms^{-1} . Given that the rotational kinetic energy of the molecule is two thirds of its translational kinetic energy. Calculate its angular velocity at s.t.p. assuming that molecular rotation takes place about an axis through the centre and perpendicular to the line joining the atoms.

Solution:

Distance between the atoms in $O_2(r) = 1.2 \times 10^{-10} \text{ m}$

At s.t.p, molecular speed (v) = 460 ms^{-1}

At s.t.p, angular velocity (ω) = ?

The M.I. of O_2 about an axis through its centre and perpendicular to the line joining them is

$$I = m \left(\frac{r}{2} \right)^2 + m \left(\frac{r}{2} \right)^2 = \frac{1}{2} mr^2;$$

Where, m = mass of oxygen atom

$$\text{Given, } (K.E.)_{\text{rot}} = \frac{2}{3} (K.E.)_{\text{trans}}$$

$$\text{or, } \frac{1}{2} I \omega^2 = \frac{2}{3} \times \frac{1}{2} (m + m) v^2$$

$$\text{or, } \frac{1}{2} \cdot \frac{1}{2} mr^2 \cdot \omega^2 = \frac{2}{3} \times \frac{1}{2} \times 2mv^2$$

$$\text{or, } \frac{mr^2 \omega^2}{4} = \frac{2}{3} mv^2$$

$$\text{or, } \omega^2 = \frac{8}{3} \frac{v^2}{r^2}$$

$$\therefore \omega = \frac{v}{r} \sqrt{\frac{8}{3}}$$

$$= \frac{460}{1.2 \times 10^{-10}} \times \sqrt{\frac{8}{3}}$$

$$= 6.3 \times 10^{12} \text{ rads}^{-1}$$

- 27.** In the design of a passenger bus, it is proposed to derive the motive power from the energy stored in a flywheel. The flywheel, which has a moment of inertia of $4.0 \times 10^2 \text{ kgm}^2$, is accelerated to its maximum rate of rotation 3.0×10^3 revolutions per minute by electric motors at stations along the bus route, (a) calculate the maximum kinetic energy which can be stored in the flywheel. (b) If, at an average speed of 36 kilometers per hour, the power required by the bus is 20 KW, what will be the maximum possible distance between stations on the level?

Solution:

$$I = 4 \times 10^2 \text{ kgm}^2$$

$$f_{\text{max}} = 3 \times 10^3 \text{ rev/min} = \frac{3 \times 10^3}{60} \text{ rev/s} = 50 \text{ rev/s.}$$

$$\text{a. } K.E._{\text{max}} = \frac{1}{2} I \omega_{\text{max}}^2 = \frac{1}{2} I (2\pi f_{\text{max}})^2 = 2\pi^2 I f_{\text{max}}^2$$

$$= 2 \times 3.14 \times 4 \times 10^2 \times (50)^2 = 2 \times 10^7 \text{ J}$$

$$\text{b. } v = 36 \text{ Kmhr}^{-1} = \frac{36 \times 1000}{60 \times 60} \text{ ms}^{-1} = 10 \text{ ms}^{-1}$$

$$P = 20 \text{ KW} = 20000 \text{ watts}$$

Maximum possible distance (s) = ?

$$\text{Since, } K.E. \text{ max} = F \times S$$

$$\text{or, } K.E._{\text{max}} = \frac{P}{v} \times S \quad [\text{since, } P = F \times v]$$

$$\text{or, } S = \frac{K.E. \text{ max} \times v}{P} = \frac{2 \times 10^7 \times 10}{20000} = 10^4 \text{ m} = 10 \text{ km}$$

- 28.** A flywheel of moment of inertia 0.32 kgm^2 is rotated steadily at 120 rad s^{-1} by a 50 W electric motor. (a) Find the kinetic energy and angular momentum of the flywheel. (b) Calculate the value of the frictional couple opposing the rotation. (c) Find the time taken for the wheel to come to rest after the motor has been switched off.

Solution:

$$I = 0.32 \text{ kgm}^2 \quad \omega = 120 \text{ rad s}^{-1}$$

$$P = 50 \text{ W}$$

$$\text{a. } K.E. = ?$$

Angular momentum (L) = ?

We have the relation,

$$K.E. = \frac{1}{2} I \omega^2 = \frac{1}{2} \times 0.32 \times 120^2 = 2304 \text{ J}$$

$$\text{and, } L = I \omega = 0.32 \times 120 = 38.4 \text{ kgm}^2 \text{s}^{-1}$$

$$\text{b. } \text{Frictional couple } (\tau) = ?$$

We have the relation,

$$P = \tau \omega$$

$$\text{or, } \tau = \frac{P}{\omega} = \frac{50}{120} = \frac{5}{12} = 0.42 \text{ Nm}$$

$$\text{c. } \text{Time to come to rest } (t) = ?$$

We have the relation,

$$t = \frac{\omega' - \omega_0}{\alpha} = \frac{0 - \omega}{\alpha} = -\frac{\omega}{\alpha}$$

$$\text{Also, } \tau = I \alpha$$

$$\text{or, } \alpha = \frac{\tau}{I} = \frac{-0.42}{0.32} = -1.31 \text{ rads}^{-2}$$

Here, τ is negative for frictional couple

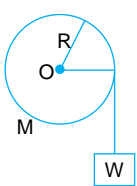
$$\therefore t = -\frac{\omega}{\alpha} = \frac{-120}{-1.31} = 91.4 \text{ sec.}$$

Exercise

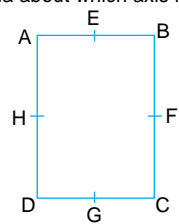
A. Multiple Choice Questions

Circle the best alternative to the following questions:

- A rotating disc has _____ kinetic energy if mass is M and velocity is V
 - MV^2
 - $\frac{1}{2}MV^2$
 - $\frac{3}{4}MV^2$
 - $\frac{MV^2}{4}$
- When the size of earth is reduced to half, mass remaining the same, the time period of earth rotation will be :
 - 6 hours
 - 24 hours
 - 12 hours
 - 48 hours
- The body applied with constant torque changes the angular momentum I_0 to final angular momentum $4I_0$ in 3 sec, the find torque
 - $3I_0$
 - I_0
 - $4I_0$
 - $2I_0$
- If the ratio of the earth's orbit is made one fourth, the duration of year will become
 - $\frac{1}{2}$ times
 - $\frac{1}{4}$ times
 - $\frac{1}{8}$ times
 - $\frac{1}{16}$ times
- If there is a change of angular momentum from 2J to 4J in 4 sec. The the torque is
 - 0.25J
 - 0.5J
 - 1J
 - 2J
- In the figure shown below, a wight W is attached to a string wrapped round a solid cylinder of mass M mounted on a frictionless horizontal axle at O . If the weight starts from rest and falls a distance h , then the speed at this instant will be

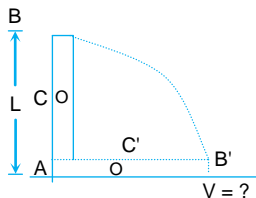


- proportional to R
 - proportional to $1/R$
 - proportional to $1/R^2$
 - independent of R
- A wheel of mass 10kg has a moment of inertia 160kgm^2 about its own axis. The radius of gyration is
 - 10m
 - 4m
 - 5m
 - 6m
- The radius of gyration of a solid disc of mass 1 kg mass and radius 50 cm about an axis through centre of mass and perpendicular to its face is
 - 25 cm
 - $25\sqrt{2}$ cm
 - 20cm
 - $25\sqrt{6}$ cm
- In rotational motion, the physical quantity that imparts angular acceleration is
 - Force
 - Torque
 - moment of inertia
 - angular momentum
- A sphere of mass 0.5 kg and diameter 1m rolls without sliding with a constant velocity 5 m/s. Calculate the fraction of total energy with rotational
 - $\frac{1}{2}$
 - $\frac{2}{7}$
 - $\frac{5}{7}$
 - $\frac{7}{10}$
- A small solid sphere rolls without slipping down a 30° inclined plane . It's acceleration is
 - $\frac{25}{7}$
 - $\frac{7}{25}$
 - $\frac{16}{9}$
 - 5
- The moment of inertia of a uniform circular disc about diameter is I the M.I. about an axis passing through a point on its rim and perpendicular to plane will be
 - $3I$
 - $4I$
 - $5I$
 - $6I$
- Two solid spheres are made of same material and having the radii in the ratio 1:2. The ratio of moment of inertia is
 - 1:2
 - 1:4
 - 1:16
 - 1:32
- Two particles of masses m_1 and m_2 are at distance x . Then, the center of mass lies at distance from m_1 :
 - $\left(\frac{m_1}{m_2}\right)x$
 - $\left(\frac{m_1}{m_1 + m_2}\right)x$
 - $\left(\frac{m_2}{m_1}\right)x$
 - $\left(\frac{m_2}{m_1 + m_2}\right)x$
- The centre of mass of thin triangular plate lies on
 - the centre of its base
 - its centroid
 - mid – point
 - one of its vertices
- An inclined plane makes an angle 30° with horizontal. A solid sphere rolling down this inclined plane from rest without slipping has a linear acceleration equal to
 - $\frac{g}{3}$
 - $\frac{2g}{3}$
 - $\frac{5}{7}g$
 - $\frac{5}{14}g$
- In a rectangle ABCD (with $BC = 2AB$) given below, the moment of inertia about which axis is minimum?



 - About BC
 - About BD
 - About HF
 - About EG

18. A rod of length L , whose lower end is pivoted, is allowed to fall; then speed of the upper end when it hits the ground is



- a. \sqrt{gL}
 b. $\sqrt{5gL}$
 c. $\sqrt{3gL}$
 d. $3\sqrt{gL}$

19. Three point masses each of mass m are located at the corners of an equilateral triangle of side a . Then the moment of inertia of this system about an axis along one side of the triangle is

- a. $\frac{3}{2} ma^2$
 b. $\frac{3}{4} ma^2$
 c. ma^2
 d. $3 ma^2$

20. In a carbon-monoxide molecule, the carbon and the oxygen atoms are separated by a distance 1.12×10^{-10} m. The distance of the centre of mass, from carbon atom is

- a. 0.48×10^{-10} m
 b. 0.51×10^{-10} m
 c. 0.56×10^{-10} m
 d. 0.64×10^{-10} m

Answers Key

1. d	2. a	3. b	4. d	5. b	6. d	7. b	8. b	9. b	10. b
11. a	12. b	13. b	14. d	15. b	16. d	17. d	18. c	19. b	20. d

B. Short Questions


- What do you mean by moment of inertia?
- What happens when a skater, spinning with arms outstretched, pulls her arms in?
- What is radius of Gyration ?
- Why is it harder to open and shut the door if we apply force near the hinge?
- If a body is rotating, is it necessarily being acted by an external torque?
- What would happen if wheels of your cycle had no spokes?
- Define angular momentum.
- State principle of conservation of angular momentum.
- A ballet dancer stretches her arms to reduce her motion. Explain. [HSEB 2058]
- A dancer girl is rotating over a turn table with her arms outstretched. If she lower her arms how does this affect her motion? [HSEB 2057]
- A ring, a disc and a sphere all of the same radius and mass roll down an incline plane from the same height h . Which of the three reach bottom first and last?
- Flywheels are used in railway engine, why?
- Two satellites of equal mass are orbiting the earth at different heights. Considering them as particles, do they have equal moment of inertia?
- How can a diver diving from a height increases the number of turns of his body in air?
- A metal ring is melted and a solid sphere is made out of it. What happens to the M.I. about a vertical axis through the centre?
- A rope dancer holds an umbrella or pole to balance him, why?
- Two metal spheres have same mass and radius. If one is hollow and the other is solid, which will have greater M.I.?
- Force A has a magnitude twice that of force B. Is it possible for B to exert a greater torque on an object than force A?
- Why is it more difficult to revolve a stone by tying it to a longer string than by tying it to a shorter string?
- Why is friction necessary for the disc to roll on the surface in the straight line?
- A ballet dancer stretches her hands when she wants to come to rest. Why? [HSEB 2070]

C. Long Questions

1. Define moment of inertia and angular momentum. Establish a relation between them. [HSEB 2068]
2. What is meant by a couple? Derive an expression for the work done by a couple. [HSEB 2066 old]
3. Define the terms couple and moment of a couple. Derive an expression for the work done by a couple. [HSEB 2062]
4. Explain the meaning of the term 'moment of inertia'. Show that the quantity $\frac{1}{2} I\omega^2$ is the kinetic energy of rotation of a rigid body rotating about an axis with angular velocity ω . [HSEB 2060]
5. Define moment of inertia. How is it related with rotational kinetic energy of a body? [HSEB 2057]
6. What is meant by moment of inertia? How is it related with the rotational kinetic energy of a body? [HSEB 2051]
7. Explain the principle of moment and moment of inertia of a rigid body. [HSEB 2062, S]
8. State and explain the principle of conservation of angular momentum with example.
9. Obtain an expression for the kinetic energy of a body rolling on a horizontal surface without slipping.
10. Show that the rate of doing work on a rotating body by a torque is equal to the product of the torque and the angular velocity of the body.
11. What is radius of gyration? Show that the acceleration of a body rolling down an inclined plane is, $a = \frac{g \sin \theta}{1 + K^2/r^2}$, where θ is, the angle of inclination of the plane, K is the radius of gyration and r is the radius of the body.
12. Show that in rotational motion, power is the product of torque and angular velocity.
13. Define moment of inertia. Obtain an expression for the moment of inertia of a thin and uniform rod about an axis passing through the centre and perpendicular to its length. [HSEB 2073 C]
14. Define moment of inertia. How is it related with angular momentum of a body rotating about an axis of rotation? [NEB 2074]
15. Explain the concept of torque and angular acceleration in the case of a rigid body. Derive a relation between them [NEB 2074]

D. Numerical Questions

1. A constant torque of 200 Nm turns a wheel about its centre. The moment of inertia about this axis is 100 kg m². Find the kinetic energy gained after 20 revolutions. [Ans: 25132.82J]
2. The angle θ through which a bicycle wheel turns is given by $\theta(t) = a + bt^2 - ct^3$; where a , b and c are positive constants such that for t in seconds, θ will be in radians. (a) Calculate the angular acceleration of the wheel as a function of time. (b) At what time is the angular velocity of the wheel instantaneously not changing? [Ans: $\alpha = 2b - 6ct$, $t = b/3c$]
3. An electric fan is turned off, and its angular velocity decreases uniformly from 500 rev/min to 200 rev/min in 4.00 s. (a) Find the angular acceleration in rev/s² and the number of revolutions made by the motor in the 4.00s interval. (b) How many more seconds are required for the fan to come to rest if the angular acceleration remains constant at the value calculated in part (a)? [Ans: $\alpha = -1.25 \text{ rev/s}^2$, $n = 23.3$, $t = 2.66 \text{ secs}$]
4. A wheel rotates with a constant angular velocity of 6.00 rad/s. (a) Compute the radial acceleration of a point 0.500 m from the axis, using the relation $a_{\text{rad}} = \omega^2 r$, (b) Find the tangential speed of the point and compute its radial acceleration from the relation $a_{\text{rad}} = \frac{V^2}{r}$. [Ans: 18.0 ms^{-2} , 3.00 ms^{-1} , 18.0 ms^{-2}]
5. The spin cycles of washing machine have two angular speeds, 423 rev/min and 640 rev/min. The internal diameter of the drum is 0.470 m. (a) What is the ratio of the maximum radial force on the laundry for the higher angular speed to that for the lower speed? (b) What is the ratio of the maximum tangential speed of the laundry for the higher angular speed to that for the lower speed? [Ans: 2.29:1, 1.51:1]

6. A roller turns through an angle $\theta(t)$ given by $\theta(t) = \gamma t^2 - \beta t^3$, where $\gamma = 3.20 \text{ rad/s}^2$ and $\beta = 0.500 \text{ rad/s}^3$. (a) Calculate the angular velocity of the roller as a function of time. (b) Calculate the angular acceleration of the roller as a function of time. (c) What is the maximum positive angular velocity, and at what value of t does it occur?
[Ans: $6.40t - 1.50t^2$; $6.40 - 3.00t$, 6.83 rad s^{-1} , 2.13 sec]
7. A 1.50 kg grinding wheel is in the form of a solid cylinder of radius 0.100 m . (a) What constant torque will bring it from rest to an angular speed of 1200 rev/min in 2.5 s ? (b) Through what angle has it turned during that time? (c) Calculate the work done by the torque. (d) What is the grinding wheel's kinetic energy when it is rotating at 1200 rev/min ?
[Ans: 0.38 Nm , 160 rad , 61 J , 60 J]
8. A solid disk is rolling without slipping on a level surface at a constant speed of 2.00 m/s . How far can it roll up a 30.0° ramp before it stops?
[Ans: 0.612 m]
9. Find the magnitude of the angular momentum of the second hand on a clock about an axis through the centre of the clock face. The clock hand has a length of 15 cm and a mass of 6 g . Take the second hand to be a slender rod rotating with constant angular velocity about one end.
[Ans: $4.71 \times 10^{-6} \text{ kg m}^2 \text{ s}^{-1}$]
10. Energy of 484 J is spent in increasing the speed of a flywheel from 60 r.p.m. to 240 r.p.m. Find the moment of inertia of the wheel.
[Ans: 1.63 kgm^2]
- 

2 CHAPTER



Christiaan Huygens
(1629-1695)

Periodic Motion

Syllabus:

Equation of simple harmonic motion (SHM). Energy in SHM. Application of SHM: vertical oscillation of mass suspended from coiled spring. Angular SHM, simple pendulum. Oscillatory motion: Damped oscillation, Forced oscillation and resonance.



Learning Objectives

After studying this chapter, students should be able to:

- ✎ Define simple harmonic motion and state its equation.
- ✎ Derive the expressions for energy in simple harmonic motion
- ✎ Derive the expression for period for vertical oscillation of a mass suspended from coiled spring
- ✎ Describe angular simple harmonic motion and find its period
- ✎ Derive expression for period of simple pendulum
- ✎ Explain the damped oscillation
- ✎ Describe forced oscillation and resonance with suitable examples
- ✎ Solve the numerical problems and conceptual questions regarding the periodic motion

2.1 Introduction

When a body moves along a straight line, then its motion is translatory motion, which we have discussed in previous chapters. In this chapter, we discuss the vibratory motion of a physical system.

2.2 Periodic Motion

The motion which repeats itself after equal intervals of time is called a **periodic motion**. The interval of time is called the **time period** of the periodic motion. For examples, the motion of the pendulum of a wall clock, oscillations of a mass suspended from a spring, the motion of a planet around the sun, the motion of the hands of a clock etc.

2.3 Oscillatory Motion

If a body moves back and forth (to and fro) repeatedly about a mean position, then it is said to possess **oscillatory** or **vibrating motion**. For examples, the motion of the pendulum of a wall clock, oscillations of a mass suspended from a spring, the oscillations of the hands of a walking person etc.

An oscillatory motion is always periodic but a periodic motion may not be oscillatory. For example, the motion of earth round the sun is periodic but not oscillatory. An oscillatory motion can be expressed in terms of sine or cosine function or their combination. So, the oscillatory motion is also called the **harmonic motion**.

2.4 Simple Harmonic Motion (SHM)

A simple harmonic motion is defined as an oscillatory motion about a fixed point (mean position) in which the restoring force is always directly proportional to the displacement from that point and is always directed towards that point.

If x be the small displacement from the mean position of a particle and F be the restoring force acting on it, then

$$F \propto x$$

$$\text{or, } F = -Kx \quad \dots (1)$$

Where K is a proportionality constant known as **force constant**. The negative sign indicates that the restoring force F is developed opposite to the displacement from the mean position.

From Newton's second law of motion.

$$F = ma \quad \dots (2)$$

Where, m = mass of the particle and a = acceleration

From equations (1) and (2), we get

$$ma = -Kx$$

$$\text{or, } a = -\frac{K}{m}x \quad \dots (3)$$

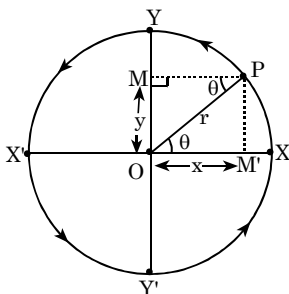
Here, K and m are constants.

$$\therefore a \propto x$$

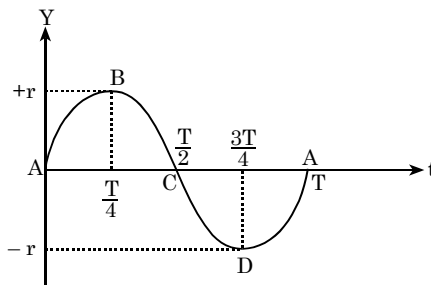
Equation (3) shows that the acceleration in SHM is always directly proportional to the displacement from the mean position and negative sign indicates that it is always directed towards the mean position.

2.5 Circle of Reference and Equations of SHM

Suppose a particle is moving with uniform speed along the circumference of a circle of radius r . The circle is called **reference circle** and the particle is called **reference particle** or **generating particle**.



[Fig. 2.1, (a) Circle of reference]



[Fig. 2.1, (b) Graphical representation of motion of particle p]

Let P be the position of the reference particle at any instant t . Let M be the foot of perpendicular drawn from the position P on the diameter YOY' as shown in figure 2.1 (a).

When the particle P moves from X to Y, then the foot of perpendicular M moves from O to Y.

When the particle P moves from Y to X', then M moves from Y to O.

Similarly, when the reference particle P moves from X' to Y', the foot M moves from O to Y'.

When P moves from Y' to X, then M moves from Y' to O.

Thus, when the reference particle P completes one revolution along the circle, the foot of perpendicular M completes one vibration about the mean position O along the diameter YOY'.

The motion of foot of perpendicular M along the diameter YOY' is **simple harmonic motion (SHM)** and the motion of particle P along the circle is **uniform circular motion**. The points A, B, C, D and A in figure 2.1(b) correspond to respective points X, Y, X', Y' and X in figure 2.1 (a).

The centre of circle O is called **mean position** and ends of diameter Y or Y' are called **extreme positions of SHM**.

A simple harmonic motion (SHM) possess the following **characteristics**:

1. **Displacement (y)**: The distance of foot of perpendicular M from the mean position O, is called displacement (y) in SHM. Let θ be the angular displacement of the reference particle P at time t as shown in reference circle 2.1(a).

Then, in $\triangle OPM$,

$$\sin\theta = \frac{OM}{OP} = \frac{y}{r}$$

$$\text{or, } y = r\sin\theta$$

$$\therefore y = r\sin\omega t \quad \dots (1)$$

Where, $\theta = \omega t$ and ω is constant angular velocity. Equation (1) is the **displacement equation** for SHM.

If the foot of perpendicular M' of the particle is taken on the diameter XOX', then

$$x = r\cos\theta$$

$$\therefore x = r\cos\omega t \quad \dots (2)$$

2. Amplitude (y_{\max} or r):

We have, the displacement equation in SHM is,

$$y = r \sin \omega t = r \sin \theta$$

Here, y is maximum if $\sin \theta = 1$.

$$\therefore y_{\max} = r$$

So, maximum displacement of the particle from the mean position is called **amplitude** or **displacement amplitude** in SHM which is equal to the radius r of the reference circle. Obviously, the displacement is maximum at extreme positions Y or Y' .

3. Velocity (v):

The velocity in SHM at an instant is defined as the rate of change of displacement at that instant. So,

$$v = \frac{dy}{dt}$$

$$\text{or, } v = \frac{d}{dt} (r \sin \omega t) \quad [\text{Since, } y = r \sin \omega t]$$

$$\text{or, } v = r \omega \cos \omega t \quad \dots (3)$$

$$\text{or, } v = r \omega \sqrt{1 - \sin^2 \omega t}$$

$$\text{or, } v = \omega \sqrt{r^2 - (r \sin \omega t)^2}$$

$$\text{or, } v = \omega \sqrt{r^2 - y^2} \quad \dots (4) \quad [\text{Since, } y = r \sin \omega t.]$$

Equation (3) and (4) are the **equations for velocity** in SHM. These equations show the velocity (v) is not uniform in SHM.

At mean position O , $y = 0$, $\therefore v = r \omega$ (maximum value)

At extreme positions Y or Y' , $y = r$, $\therefore v = 0$ (minimum value)

So, a particle in SHM has **maximum velocity** at mean position and **minimum velocity** (i.e. zero) at the extreme positions.

4. Accelerations (a):

The rate of change of velocity is called acceleration (a). So,

$$a = \frac{dv}{dt} = \frac{d}{dt} (r \omega \cos \omega t) \quad [\text{Since, } v = r \omega \cos \omega t]$$

$$\text{or, } a = r \omega \frac{d}{dt} (\cos \omega t) = r \omega (-\sin \omega t) \omega$$

$$\text{or, } a = -\omega^2 (r \sin \omega t)$$

$$\therefore a = -\omega^2 y \quad \dots (5) \quad [\text{Since, } y = r \sin \omega t, \text{ from equation (1)}]$$

This is the **equation for the acceleration** in SHM. This equation (5) shows that acceleration in SHM is directly proportional to the displacement from the mean position and negative sign shows that it is always directed towards the mean position.

At the mean position O , $y = 0$,

$$\therefore a = 0 \text{ (minimum value)}$$

At extreme positions Y or Y' , $y = r$,

$$\therefore a = -\omega^2 r \text{ (maximum value)}$$

Thus, a particle in SHM has **zero acceleration** at the mean position and **maximum acceleration** at the extreme positions.

5. Time period (T):

The time taken by the particle to complete one oscillation is called time period in SHM (T).

The magnitude of acceleration in SHM is given by,

$$a = \omega^2 y$$

$$\text{or, } \omega^2 = \frac{a}{y}$$

$$\text{or, } \omega = \sqrt{\frac{a}{y}}$$

$$\text{or, } \frac{2\pi}{T} = \sqrt{\frac{a}{y}}$$

$$\text{or, } T = 2\pi \sqrt{\frac{y}{a}}$$

$$\therefore T = 2\pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}} \quad \dots (6)$$

In this time T , the foot of perpendicular M makes one complete oscillation along the diameter YOY' i.e. from O to Y , back to O , then to Y' and finally again back to O .

6. **Frequency (f)** : The number of oscillations made in one second is called **frequency (f)** in SHM.

In T seconds, the particle makes 1 oscillation.

In 1 second, the particle makes $\frac{1}{T}$ oscillations

$$\therefore \text{frequency} = \frac{1}{\text{Time period}}$$

$$\text{or, } f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{\text{acceleration}}{\text{displacement}}} \quad \dots (7)$$

7. **Wavelength (λ)**: The linear distance travelled by the particle in one oscillation is called **wavelength (λ)** of the particle executing SHM. If T be the time period of oscillation and v be the velocity, then

$$\lambda = vT \quad \dots (8)$$

8. **Phase**: The phase of a particle executing SHM at any instant gives the **position and direction of motion** of the particle with respect to its mean position. It is measured in terms of fraction of time period T or the fraction of angle 2π .

In the equation, $y = r \sin(\omega t + \phi_0)$,

$(\omega t + \phi_0) \rightarrow$ phase of the particle in SHM,

$\phi_0 \rightarrow$ phase constant or phase angle.

ϕ tells us the position from where the time was considered.

Let, $\omega t + \phi_0$ be denoted by ϕ .

Then, $\phi = \omega t + \phi_0$

or, $\phi - \phi_0 = \omega t$

So, phase change in time t is, $\phi - \phi_0 = \omega t = \left(\frac{2\pi}{T}\right)t$

The phase change in time T is, $\phi - \phi_0 = \left(\frac{2\pi}{T}\right)T = 2\pi$.

This shows that the phase change in T seconds will be 2π radian which actually means 'no change

in phase'. Therefore, the **time period** (T) can also be defined as the time interval during which the phase of the vibrating particle changes by 2π .

9. Graphical Representation of Displacement, Velocity and Acceleration in SHM:

The displacement (y), velocity (v) and acceleration (a) equations in SHM are,

$$y = r \sin \omega t = r \sin \left(\frac{2\pi}{T} \right) t, \quad v = r \omega \cos \omega t = r \omega \cos \left(\frac{2\pi}{T} \right) t$$

$$\text{and, } a = -\omega^2 y = -\omega^2 r \sin \omega t = -\omega^2 r \sin \left(\frac{2\pi}{T} \right) t$$

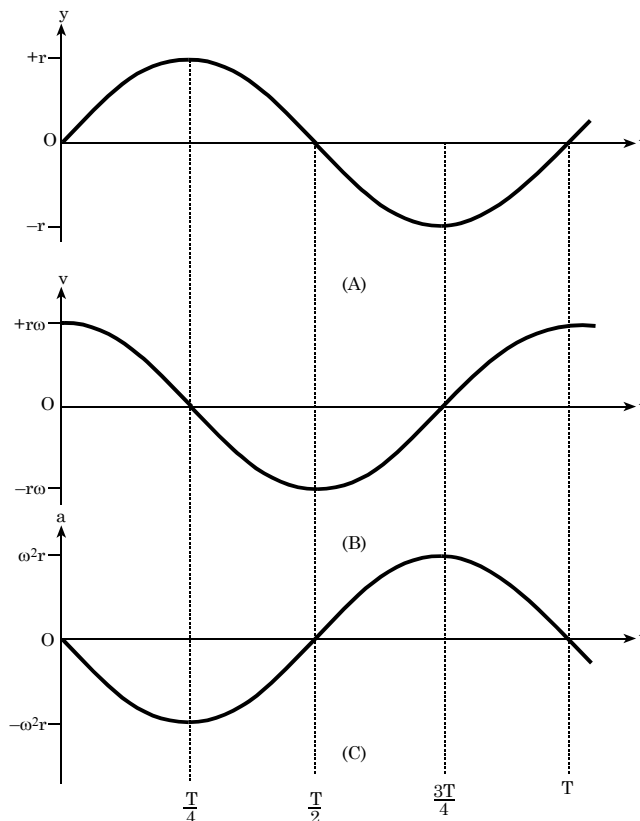
$$\text{At } t = 0, y = 0, v = r\omega \text{ and } a = 0$$

$$\text{At } t = \frac{T}{4}, y = r, v = 0 \text{ and } a = -\omega^2 r$$

$$\text{At } t = \frac{T}{2}, y = 0, v = -r\omega \text{ and } a = 0$$

$$\text{At } t = \frac{3T}{4}, y = -r, v = 0 \text{ and } a = \omega^2 r$$

$$\text{At } t = T, y = 0, v = r\omega \text{ and } a = 0$$



[Fig. 2.2, Graphical representation of (a) Displacement, (b) Velocity and (c) Acceleration in SHM]

2.6 Energy in Simple Harmonic Motion

A particle executing SHM has both kinetic energy and potential energy. The particle possesses K.E. by virtue of its motion and P.E. as a restoring force acts on it when displaced from equilibrium (mean)

position. Since, both the velocity and position of the oscillating particle are changing continuously with time, but the total energy remains conserved.

Kinetic Energy (E_K)

If m and v be the mass and velocity of the particle executing SHM respectively, then

$$E_K = \frac{1}{2} mv^2$$

Since, $v = \omega\sqrt{r^2 - y^2}$

$$\therefore E_K = \frac{1}{2} m\omega^2(r^2 - y^2) \quad \dots (1)$$

Where, r = amplitude of SHM and ω = angular velocity

Potential Energy (E_p)

If y be the displacement of the particle from mean position, then the acceleration is,

$$a = -\omega^2 y$$

The restoring force F on the particle is,

$$F = ma = m(-\omega^2 y)$$

$$\text{or, } F = -m\omega^2 y$$

Small work done ' dW ' to further displace the particle through small displacement ' dy ' against the restoring force F is,

$$dW = -F dy$$

The negative sign shows that restoring force F acts opposite to the displacement from mean position.

$$\text{or, } dW = -(-m\omega^2 y) dy$$

$$\text{or, } dW = m\omega^2 y dy$$

The total work done to displace the particle through displacement y from mean position is,

$$W = \int dW = \int m\omega^2 y dy = m\omega^2 \int_0^y y dy = m\omega^2 \left[\frac{y^2}{2} \right]_0^y$$

$$\therefore W = \frac{1}{2} m\omega^2 y^2$$

This work done is stored in the particle in the form of its potential energy (E_p). Thus,

$$E_p = \frac{1}{2} m\omega^2 y^2 \quad \dots (2)$$

Total Energy (E)

The total energy E of the particle executing SHM is,

$$E = E_K + E_p$$

$$\text{or, } E = \frac{1}{2} m\omega^2(r^2 - y^2) + \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2 r^2 - \frac{1}{2} m\omega^2 y^2 + \frac{1}{2} m\omega^2 y^2$$

$$\therefore E = \frac{1}{2} m\omega^2 r^2 \quad \dots (3)$$

Here, m , ω and r are constants with time. Hence, equation (3) shows that **total energy of a particle executing SHM remains constant**.

Special Cases:**1. At mean position, $y = 0$**

$$\therefore E_k = \frac{1}{2} m\omega^2(r^2 - y^2) = \frac{1}{2} m\omega^2(r^2 - 0^2) = \frac{1}{2} m\omega^2 r^2$$

$$\text{and, } E_p = \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2 (0)^2 = 0$$

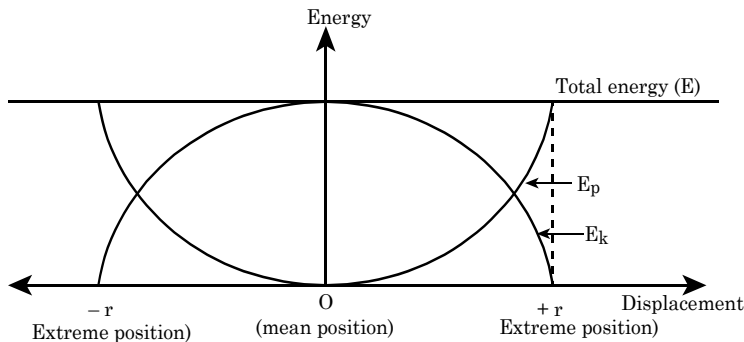
So, at mean position, P.E. is zero and K.E. is maximum. So, total energy of the particle executing SHM at mean position is in the form of kinetic energy.

2. At extreme position, $y = r$

$$\therefore E_k = \frac{1}{2} m\omega^2(r^2 - y^2) = \frac{1}{2} m\omega^2(r^2 - r^2) = 0$$

$$\text{and, } E_p = \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2 r^2$$

So, K.E. is zero and P.E. is maximum at extreme position. So, total energy is in the form of P.E. at extreme position of a SHM.



[Fig. 2.3, Variation of kinetic energy, potential energy and total energy as a function of displacement: graphical representation of energy in SHM]

2.7 Applications of Simple Harmonic Motion

1. Oscillation of a Mass in Horizontal Spring

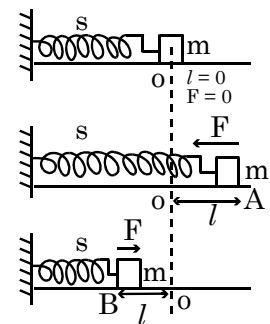
Suppose, one end of a massless spring **S** is attached to a rigid support and other free end to a mass **m**. Let the mass spring system lies horizontally on a frictionless table as shown in figure 2.4. Let **O** be the mean position of the mass corresponding to natural length of the spring. At this position **O**, the force on the mass $F = 0$.

Let the mass is pulled right to the position **A** through a distance ***l*** and ***F*** be the restoring force developed in the spring. Then, from **Hooke's law**,

$$F \propto l$$

$$\text{or, } F = -Kl \quad \dots (1)$$

Where, **K** is **force constant or spring constant** of the spring. The negative sign indicates that the restoring force acts opposite to the



[Fig. 2.4, Horizontal oscillation of a mass-spring system]

displacement. When the mass **m** is released at point **A**, it moves towards the mean position **O** where it gains kinetic energy. The mass then moves left from mean position **O** and reached to the position **B**. The spring **S** is compressed and restoring force **F** on the mass acts opposite to the direction of motion i.e., towards the mean position **O**. The mass **m** keeps on oscillating horizontally about the mean position **O**.

If '**a**' be the acceleration produced on the mass, then from Newton's second law,

$$F = ma \quad \dots (2)$$

From equations (1) and (2), we get

$$ma = -Kl$$

$$\text{or, } a = -\left(\frac{K}{m}\right)l \quad \dots (3)$$

For a mass spring system, $\frac{K}{m}$ is a constant.

$$\therefore a \propto l$$

Equation (3) shows that, for a horizontal mass spring system, acceleration is directly proportional to the displacement and the negative sign indicates that it is always directed towards the mean position. So, **motion of a horizontal mass spring system is simple harmonic motion (SHM)**.

Since, for a simple harmonic motion,

$$a = -\omega^2 l \quad \dots (4)$$

Where l = extension i.e. y .

Comparing equations (3) and (4), we get

$$\omega^2 = \frac{K}{m}$$

$$\text{or, } \omega = \sqrt{\frac{K}{m}}$$

$$\text{or, } \frac{2\pi}{T} = \sqrt{\frac{K}{m}}$$

$$\therefore T = 2\pi\sqrt{\frac{m}{K}} \quad \dots (5)$$

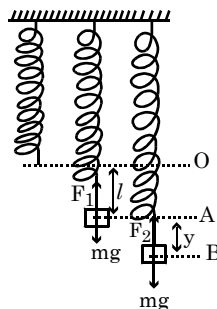
Equation (5) is the required expression for time period of oscillation, which shows that it depends on mass m and spring constant K .

2. Vertical Oscillation of a Mass Suspended from Coiled Spring

Let a mass **m** is suspended on a rigid support by the help of a massless spring as shown in figure 2.5. When the mass **m** is attached to the lower end of the spring, then it elongates through a distance l . The restoring force produced on the spring is,

$$F_1 = -Kl \quad \dots (1)$$

Here, $F_1 = mg$, and K is **force constant** of the spring. Now, the spring is at the position **A** which is the **equilibrium** or **mean position** of the loaded spring.



[Fig. 2.5, Vertical oscillation of a mass-spring system]

Let the mass is pulled down to the position B through a small distance y , then the restoring force F_2 developed on the spring is,

$$F_2 = -K(l + y) \quad \dots (2)$$

Now, if the mass is released from position B, then the resultant restoring force F which causes the oscillation is,

$$F = F_2 - F_1$$

$$\text{or, } F = -K(l + y) + Kl$$

$$\therefore F = -Ky \quad \dots (3)$$

If 'a' be the acceleration produced on the mass, then from Newton's second law,

$$F = ma \quad \dots (4)$$

From equations (3) and (4), we get

$$\text{or, } ma = -Ky$$

$$\text{or, } a = -\left(\frac{K}{m}\right)y \quad \dots (5)$$

Where, $\frac{K}{m}$ is a constant. Hence, the motion of a vertical mass spring system (loaded spring) is simple harmonic motion (SHM).

Since, for a SHM,

$$a = -\omega^2 y \quad \dots (6)$$

Comparing equations (5) and (6), we get

$$\omega^2 = \frac{K}{m}$$

$$\text{or, } \omega = \sqrt{\frac{K}{m}}$$

$$\text{or, } \frac{2\pi}{T} = \sqrt{\frac{K}{m}}$$

$$\therefore T = 2\pi\sqrt{\frac{m}{K}} \quad \dots (7)$$

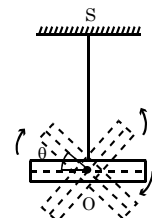
This is the required expression of time period (T) of oscillation of a vertical mass spring system. This shows that the time period of oscillation depends on mass m and spring constant K , but it is independent with value of acceleration due to gravity (g).

3. Angular Simple Harmonic Motion

Simple harmonic motion is also found in rotational motion. For example: the motion of balanced wheel of a mechanical watch, the oscillation of a heavy rod suspended from a rigid support at its centre of gravity with the help of a spring in a horizontal plane etc. In these examples as shown in figure 2.6, a restoring torque τ is developed in the spring, which is directly proportional to the angular displacement θ . That is,

$$\tau \propto \theta$$

$$\text{or, } \tau = -K\theta \quad \dots (1)$$



[Fig. 2.6, Oscillating rod in a horizontal plane about a vertical axis]

Where, K is **torsion constant** of the spring.

Since, $\tau = I\alpha$... (2)

Where, I = moment of inertia about the axis of rotation

and α = angular acceleration

From equations (1) and (2), we get

$$I\alpha = -K\theta$$

$$\therefore \alpha = -\left(\frac{K}{I}\right)\theta \quad \dots (3)$$

Here, $\left(\frac{K}{I}\right)$ is a constant for a rotating body about the given axis of rotation. Hence, the angular motion of such a body (e.g. balance wheel of a mechanical watch) is said to be the angular SHM.

The equation (3) is in the form of equation of angular SHM,

$$\alpha = -\omega^2\theta \quad \dots (4)$$

Comparing equations (3) and (4), we get

$$\omega^2 = \frac{K}{I}$$

$$\text{or, } \omega = \sqrt{\frac{K}{I}}$$

If T be the time period of oscillation, then

$$\omega = \frac{2\pi}{T}$$

$$\text{or, } \frac{2\pi}{T} = \sqrt{\frac{K}{I}}$$

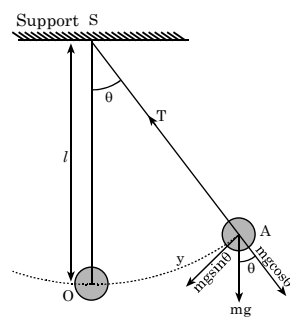
$$\therefore T = 2\pi\sqrt{\frac{I}{K}} \quad \dots (5)$$

This equation (5) is the expression for time period of oscillation which shows that T is independent with the amplitude (angle of rotation) of the motion.

4. Simple Pendulum

A simple pendulum is a heavy point mass suspended by a weightless, inextensible and flexible string from a rigid support which can oscillate freely in a vertical plane. In actual practice, it consists of a metallic bob suspended from a light cotton thread whose one end is fixed to a rigid support. The point on the rigid support by which the string is suspended is called **point of suspension**. The distance between the point of suspension and the centre of gravity of the pendulum bob is called the **effective length of simple pendulum**.

Consider a simple pendulum having mass of the bob m and effective length l as shown in figure 2.7. Point S is the **point of suspension** and point O is the **equilibrium** or **mean position** of the pendulum. Here, A represents the displaced position of the pendulum at any time such that, $\angle ASO = \theta$ is the **angular displacement** and arc $OA = y$ is the **linear displacement** of the bob.



[Fig. 2.7, Simple Pendulum]

In the displaced position A, the forces acting on the pendulum bob are:

1. **weight mg** of the bob acting vertically downward, and
2. **tension T** in the string along its length and directed towards the point of suspension S.

The weight mg of the bob can be resolved in to **two rectangular components**:

- a. **$mg\cos\theta$** opposite to the tension T along the string, and
- b. **$mg\sin\theta$** perpendicular to the string i.e., along the mean position O.

The **$mg\cos\theta$** component is balanced by the tension T in the string and **$mg\sin\theta$** component acts towards the mean position O which provides restoring force to the pendulum and tries to bring the pendulum bob back to the mean position O. So, restoring force F is,

$$F = -mg\sin\theta \quad \dots (1)$$

The negative sign indicates the opposing nature of restoring force. If 'a' is the acceleration of the bob, then

$$F = ma \quad \dots (2)$$

From equations (1) and (2), we get

$$ma = mg\sin\theta$$

$$\text{or, } a = -g\sin\theta \quad \dots (3)$$

For small θ , $\sin\theta \approx \theta$

$$\therefore a = -g\theta \quad \dots (4)$$

$$\text{Since, } \theta = \frac{\text{arc OA}}{\text{radius OS}} = \frac{y}{l}$$

So, from equation (4), we get

$$a = -g \frac{y}{l}$$

$$\text{or, } a = -\left(\frac{g}{l}\right)y \quad \dots (5)$$

For a pendulum at a given place $\left(\frac{g}{l}\right)$ is a constant.

$$\therefore a \propto y$$

Equation (5) shows that acceleration of a simple pendulum is directly proportional to the displacement from the mean position and negative sign shows that it is always directed towards the mean position.

So, **the motion of a simple pendulum is simple harmonic motion (SHM)**.

We know, for a simple harmonic motion,

$$a = -\omega^2 y \quad \dots (6)$$

Comparing equations (5) and (6), we get

$$\omega^2 = \frac{g}{l}$$

$$\text{or, } \omega = \sqrt{\frac{g}{l}}$$

$$\text{or, } \frac{2\pi}{T} = \sqrt{\frac{g}{l}}, \text{ where } T \text{ is time period.}$$

$$\therefore T = 2\pi\sqrt{\frac{l}{g}} \quad \dots (7)$$

Equation (7) is the expression for time period (T) of a simple pendulum. This equation shows that the time period of a simple pendulum is independent with the amplitude of oscillation and mass of the bob, but depends on its **effective length** and the **value of g** at a place.

Second's pendulum:

A simple pendulum whose time period is two seconds is called a **second's pendulum**.

$$\text{Since, } T = 2\pi\sqrt{\frac{l}{g}}$$

For a second's pendulum, $T = 2$ seconds.

$$\therefore 2 = 2\pi\sqrt{\frac{l}{g}}$$

$$\text{Squaring, } 4 = 4\pi^2\left(\frac{l}{g}\right)$$

$$\text{or, } l = \frac{g}{\pi^2} \quad \dots (8)$$

At sea level $g = 9.8\text{m/s}^2$.

$$\therefore l = \frac{9.8}{3.14^2} = 0.993\text{m} = 99.3\text{cm} \approx 100\text{ cm}$$

So, the length of a second's pendulum is about 100cm at sea level.

Drawbacks of Simple Pendulum

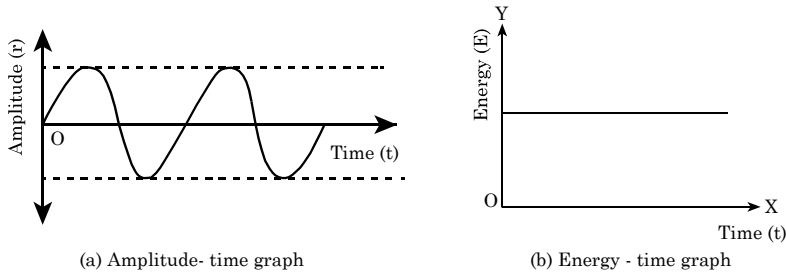
- The point of suspension may not be perfectly rigid.
- The heavy point mass can't be realized in practice.
- The weightless and perfectly inextensible string can't be available in practice.
- For large angular displacement, the formula for time period doesn't hold true.
- The motion may not be perfectly linear.

2.8 Free and Damped Oscillations

Under the action of an external periodic force, everybody in this universe has a characteristic tendency to oscillate. The frequency and amplitude of the oscillation depend on the **inertia**, **dimensions** and **elastic properties** of the body. This frequency of oscillation is called **natural frequency** of the body.

Free Oscillation

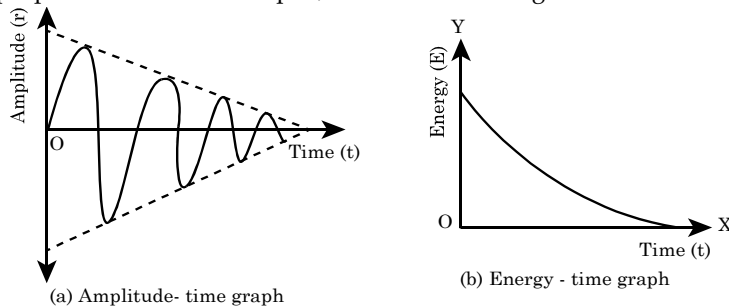
When a body capable of oscillating is slightly disturbed from its equilibrium position and let free, then it starts oscillating with its own natural frequency and with constant amplitude. The oscillation of the body in the absence of external resistive forces is called free oscillation. For example. the oscillation of a simple pendulum, tuning fork or a string etc in vacuum. The amplitude of oscillation and energy remain constant in such a free oscillation of a body as shown in figure 2.8.



[Fig. 2.8, Free oscillation]

Damped Oscillation

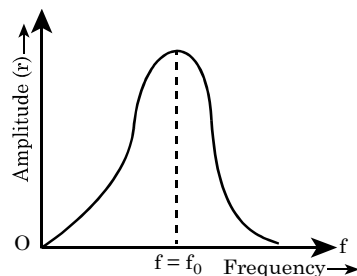
In the absence of external dissipative forces, a body executing SHM always keeps on oscillating with constant amplitude and with constant energy. This is an ideal concept only. In actual practice, there exists some dissipative forces such as friction or viscous force and these forces try to offer resistance to the motion. As a result, the amplitude and energy of the vibrating body gradually decreases with time, and finally the body comes to rest. **The gradual decrease in the amplitude and energy of the vibrating body due to dissipative forces is called damping and the oscillation is called damped oscillation.** For example, oscillation of a simple pendulum in air or a liquid, vibration of a tuning fork in air etc as shown in figure 2.9.



[Fig. 2.9, Damped oscillation]

2.9 Forced Oscillation and Resonance

The oscillations in reality are damped oscillations due to the presence of resistive forces (dissipative forces). Some external periodic force has to be applied to have **sustained oscillation** in a body. If the energy loss by the oscillating body due to damping is equal to the energy supplied by the external periodic force, then the amplitude of oscillation remains constant. Such an oscillation of a body under the influence of an external periodic force is called **forced oscillation**. In forced oscillation, the amplitude of oscillation remains constant but the body oscillates with the frequency of the external source. For example, the sound boards of stringed musical instruments suffer forced oscillations as shown in figure 2.10.



[Fig. 2.10, Amplitude-frequency curve and resonance]

Resonance

When the frequency of the external periodic force becomes equal to the natural frequency (f_0) of the oscillating body, then the body starts to oscillate with maximum amplitude. This kind of oscillation of a body is called **resonant oscillation** and the phenomenon is called **resonance**.

Figure 2.10 shows amplitude frequency curve and the point on frequency axis corresponding to maximum amplitude of oscillation gives the value of resonant frequency (f_0) or the natural frequency of the oscillating body.

Examples of Resonance

1. **The marching soldiers are ordered to break their steps while crossing a bridge:** To avoid destruction of bridge due to resonance vibration, the soldiers are ordered to break their steps while crossing the bridge. If they do not break their steps, the frequency of marching may coincide with the natural frequency of the bridge and the bridge may have in resonance vibration. Due to resonance, the maximum displacement occurs and the displacement may exceed the elastic limit of the bridge. This may cause destruction of the bridge.
2. The **rattling** produced in a car at a particular speed is due to resonance between engine and body of the car.
3. **Tuning** of a radio to a station.
4. **A distant explosion can break the glass window, due to resonance .** The resonance occurs when the frequency of sound matches with the natural frequency of the glass.

Boost for Objectives

- ☞ The graph of displacement (y), velocity (v), acceleration, force, momentum with time all are sine curves in SHM.
- ☞ Force constant of a spring depends on
 - i. Natural length of spring $K \propto \frac{1}{l}$
 - ii. Cross, sectional area : $K \propto A$
 - iii. Young's modulus of its material; $K \propto Y \therefore K = \frac{YA}{l}$
- ☞ Time period of a body oscillating from a spring is independent of g .
- ☞ The necessary and sufficient condition for SHM is proportionality between restoring force and displacement i.e. $F \propto -y$.
- ☞ In SHM, displacement is always directed away from the mean position.
- ☞ In SHM, acceleration changes both in magnitude and direction.
- ☞ A system exhibiting SHM must have elasticity as well as inertia.
- ☞ The motion of a particle describing uniform circular motion is periodic but not simple harmonic.
- ☞ Displacement and Work done by simple pendulum in one complete oscillation is zero, while distance = $4r$ (r is amplitude).
- ☞ For a particle in SHM, its time period, frequency, angular frequency, total energy and initial phase i.e. epoch remains constant with time and position.
- ☞ For a particle in SHM, its displacement, velocity, acceleration, force, PE, KE vary with time and position.
- ☞ Second pendulum is the simple pendulum with time period 2sec and length of 99.29 cm.
- ☞ If the pendulum clock is taken above and below the earth surface T increases and clock loses time.
- ☞ A girl is swinging on a swing in sitting position. When she stands up, T decreases but when another friend sits beside her on the swing, T remains the same.
- ☞ Change in amplitude doesn't change T of simple pendulum.
- ☞ The change in time period of a simple pendulum due to change in length caused by change in temperature is given by $\frac{\Delta T}{T} = \frac{1}{2} \alpha \Delta \theta$. Where, α = coefficient of linear expansion. $\Delta \theta$ = Change in temperature.

- i. In summer $\Delta\theta$ increases; T increases and the clock runs slow (loses time).
 - ii. In winter, $\Delta\theta$ decreases; T decreases and the clock runs fast (gains time)
- ☞ A tunnel has been dug through the centre of earth and a ball is released in it. It executes SHM with time period of 84.6 min and time taken to reach the centre of earth is 21 min.
 - ☞ A hollow sphere is filled with water. It is hung by a long thread. As water flows out of a hole at its bottom, the period of oscillation will first increases and then decreases till the sphere is empty.
 - ☞ A particle executing SHM with a frequency f . Its KE oscillates with frequency $2f$.
 - ☞ A SHM has frequency f , then frequency of total energy zero.
 - ☞ K.E. and P.E. oscillate with double the frequency and half the time period of SHM.
 - ☞ The phase difference between K.E and P.E. of a particle in SHM is $\pi/2$.
 - ☞ The graph between length and time period of simple pendulum is a parabola.
 - ☞ The graph between square of period and the length of simple pendulum is a straight line.
 - ☞ P.E. of a particle executing SHM is maximum at extreme position i.e., when displacement is equal to amplitude.
 - ☞ KE of a particle executing SHM is maximum at mean position i.e., when displacement is 0.
 - ☞ When SHM oscillates with a constant amplitude which does not change with time, its oscillations are called undamped oscillations.
 - ☞ When a simple harmonic system oscillates with a decreasing amplitude with time, its oscillations are called damped oscillations.
 - ☞ In damped oscillations, the frequency of oscillation decreases, whereas time period increases.



Short Questions with Answers

1. Why are soldiers ordered to break steps while crossing a bridge?
 - ☞ When frequency of steps becomes equal to the natural frequency of the bridge, then the resonance occurs and the bridge starts to oscillate with maximum amplitude. This may collapse the bridge. Hence, the soldiers are ordered to break steps while crossing a bridge.
2. Why are bells made of metal and not of wood?
 - ☞ The damping occurs quickly in wood than in metals. Due to this, wood doesn't vibrate for a long time but the metal bells vibrate for a long time. Therefore, the bells are made of metal.
3. A pendulum is taken to moon. Will it gain or lose time?
 - ☞ The time period of a pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. When the pendulum is taken to moon, then the value of g decreases and hence value of T increases. Hence, the pendulum clock loses the time.
4. On what factors does the period of a simple pendulum depend?
 - ☞ The time period (T) of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$, where l is effective length and g is acceleration due to gravity. Thus, T depends on values of l and g such that, $T \propto \sqrt{l}$ and $T \propto \frac{1}{\sqrt{g}}$.
5. If a pendulum clock is taken to a mountain top, does it gain or lose time?
 - ☞ We have, $T = 2\pi\sqrt{\frac{l}{g}}$
When the pendulum clock is taken to mountain top, then the value of g is decreased due to increase in altitude. Thus, the value of T increases and the clock loses the time.
6. A man sitting on swing stands up. What will be the effect on the periodic time of the swing?
 - ☞ Since, $T = 2\pi\sqrt{\frac{l}{g}}$. When the man stands up the centre of mass (c.m) is raised upward and hence the effective length (l) of the pendulum decreases. Due to this, the time period (T) of the swing decreases.
7. Why does the amplitude of an oscillating pendulum go on decreasing?
 - ☞ When the pendulum oscillates, air resistance opposes its motion, The kinetic energy of the pendulum is dissipated in overcoming the air resistance and its motion is damped. So, the amplitude of the pendulum goes on decreasing.
8. Is it possible that velocity is zero but acceleration is maximum?
 - ☞ Yes. At extreme position of SHM, the velocity is zero but acceleration is maximum.

9. Is it possible that acceleration is zero but velocity is maximum?

- ✎ Yes, at mean position of SHM, the acceleration is zero but velocity is maximum.

10. What is simple harmonic motion (S.H.M) ?

- ✎ A S.H.M. is a periodic motion in which acceleration of a particle is directly proportional to displacement and is always directed towards the mean position. For example, vibrations of a simple pendulum, vibrations of a tuning fork, oscillations of a freely suspended magnet in a uniform magnetic field etc.

11. What do you understand by a second's pendulum?

- ✎ A simple pendulum having time period 2 seconds is called a second's pendulum. We have, $T = 2\pi\sqrt{\frac{l}{g}}$, and $T = 2$ sec for a second pendulum on earth.

12. If the length of second's pendulum is increased further by 200 percent, will it lose or gain time? [NEB 2074]

- ✎ The time periods (T) of a simple pendulum having effective length l is, $T = 2\pi\sqrt{\frac{l}{g}}$.

For a second pendulum, $T = 2$ secs.

Here, $T \propto \sqrt{l}$. So, if length of second's pendulum is increased further by 200% then the time period of the pendulum increases. This means the pendulum takes more time to make one oscillator. Hence, pendulum will lose time.

13. A pendulum clock is in an elevator that descends at constant velocity. Does it keep correct time? If the same clock is in an elevator in free fall, does it keep correct time? [HSEB 2066 Old Question]

- ✎ This time period of a pendulum clock is, $T = 2\pi\sqrt{\frac{l}{g}}$.

When the elevator descends at constant velocity, the effective value of acceleration due to gravity ' g ' is constant neglecting variation of g with altitude. Hence the clock keeps correct time. When the elevator falls freely, then the effective value of acceleration due to gravity is zero. Hence the time period becomes infinite and the clock doesn't keep correct time.

14. A body is moving in circular path with constant speed. Is this motion a simple harmonic? Why? [HSEB 2066]

- ✎ The motion of a body in a circular path with constant speed is a periodic motion. The magnitude of acceleration is constant and the body never comes at rest. The motion is not oscillatory also.

But in SHM, the acceleration should be directly proportional to displacement and, its value is maximum

at extreme position and zero at mean position. Hence, the motion of a body in circular path with constant speed is not simple harmonic.

15. What are the drawbacks of simple pendulum?

[HSEB 2059]

- ✎ Followings are the drawbacks of a simple pendulum.
- No heavy point mass as we assume.
 - The suspension point may not be perfectly rigid.
 - No string is weightless and perfectly inextensible.
 - The amplitude of vibration may be large.
 - The effective length can not be calculated exactly.

16. If length of a simple pendulum increased by 4 times its original length, will its time period change? If yes, by how much? [HSEB 2052]

- ✎ The time period of a simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \dots (1)$$

Let, T_1 be the time period when its length becomes $4l$.

$$\text{Then, } T_1 = 2\pi\sqrt{\frac{4l}{g}}$$

$$\text{or, } T_1 = 2 \times 2\pi\sqrt{\frac{l}{g}}$$

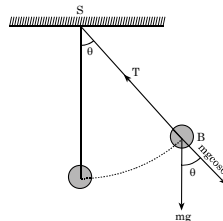
$$\therefore T_1 = 2T$$

Hence, the new time period (T_1) will be double.

17. For an oscillating simple pendulum, does the tension in the string remain constant throughout the oscillation? If not, when is it (a) minimum and (b) maximum?

- ✎ No. The tension at different points of oscillation is different. The tension is, $T = mg\cos\theta$.

At extreme position $\theta = 90^\circ$ (maximum) and so, $T = 0$ (minimum). At mean position $\theta = 0^\circ$ and so, $T = mg$ (maximum).



18. Why do we say that velocity and acceleration of a particle executing SHM are out of phase?

- ✎ Since, in SHM, velocity (v) and acceleration (a) are given by, $v = r\omega\cos\omega t$

$$\text{and, } a = -\omega^2 y = -\omega^2 r\sin\omega t = r\omega^2\cos(\omega t + \frac{\pi}{2}).$$

The phase difference between v and a is $\frac{\pi}{2}$. So v and a are out of phase in SHM.

19. A hollow bob of simple pendulum with a small hole at its bottom is filled with water. As the water slowly flows out, how does time period vary?

✎ We have, $T = 2\pi \sqrt{\frac{l}{g}}$. At first, the effective length of simple pendulum increases due to which the time period increases. Later, the effective length of it decreases due to which its time period also decreases.

20. Suppose a hole is dug in the earth through its centre. A ball is dropped in to the hole. Will the ball return to the thrower? Explain.

✎ We know, value of g is maximum at the surface and minimum at the centre of earth. So, the motion of the ball along the hole is SHM. Due to this reason, the motion is to and fro along the hole and will return to the thrower.

21. Distinguish between periodic motion and oscillatory motion.

✎ The motion, which repeats itself after equal interval of time, is called a **periodic motion**. If a body moves back and forth (to and fro) repeatedly about a mean position, then it is said to possess **oscillatory motion**. For example, the revolution of a planet around the sun is a periodic motion but not an oscillatory motion. Therefore, **all oscillatory motion are periodic but all periodic motion may not necessarily be oscillatory**.

22. A clock based on an oscillatory spring is taken to the moon. What will be the time period there?

✎ For an **oscillatory spring**, time period (T) is $T = 2\pi \sqrt{\frac{m}{K}}$. Where m is the mass attached and K is the spring constant. Here, both m and K are constants. For an oscillatory spring, the time period (T) is independent with the value of ' g '. Therefore, the time period (T) remains same on the surface of the moon as it is on earth's surface.

23. The bob of a simple pendulum is negatively charged. A positively charged metal plate is brought just below and the pendulum is allowed to oscillate. How does it affect the time period?

✎ When the negatively charged bob oscillates the positively charge plate exerts a downward force to the bob all the time. This increases the effective downward acceleration.

The time period (T) becomes

$$T = 2\pi \sqrt{\frac{l}{g_{\text{total}}}} = 2\pi \sqrt{\frac{l}{g + a}}$$

Here, $g + a > g$. So, the new time period will be less and the pendulum becomes faster or gains time.

24. To double the total energy for an oscillatory mass-spring system, by what factor must the amplitude increase? What effect does this change have on the frequency?

✎ Total energy (E) of SHM is, $E = \frac{1}{2} m \omega^2 r^2$... (1)

If r_1 be the new amplitude for double energy, then,

$$2E = \frac{1}{2} m \omega^2 r_1^2 \quad \dots (2)$$

Dividing equation (2) by equation (1), we get

$$\frac{2E}{E} = \frac{r_1^2}{r^2} \Rightarrow r_1^2 = 2r^2 \Rightarrow r_1 = \sqrt{2} r$$

So, to double energy, amplitude should increase by $\sqrt{2}$ times.

Also, frequency (f) is

$$f = \frac{1}{2\pi} \sqrt{\frac{K}{m}} \quad \dots (3)$$

This shows that f is independent with the amplitude (r) of motion. Hence frequency (f) remains constant.

25. The position of a certain object in SHM is given as a function of time by $x = (0.050\text{m}) \cos [(290 \text{ rad/s})t + (2.5\text{rad})]$. What are the amplitude, period, phase angle and the initial position for this motion?

✎ Given equation is,

$$x = (0.050\text{m}) \cos [(290 \text{ rad/s})t + (2.5 \text{ rad})] \quad \dots (1)$$

Comparing this equation with standard equation of SHM,

$$x = r \cos [\omega t + \phi] \quad \dots (2)$$

Amplitude (r) = 0.050m.

$\omega = 290 \text{ rad/s}$

We know that,

$$T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{290} = 0.02 \text{ sec}$$

So, period (T) = 0.02 sec

Also, phase angle (ϕ) = 2.5 rad

When $t = 0$, then initial position x_0 is

$$x_0 = (0.050\text{m}) \cos [2.5\text{rad}]$$

$$\text{or, } x_0 = (0.050\text{m}) \times (-0.801) = -0.04\text{m.}$$

26. What do you understand by a second's pendulum? If it is taken to moon, will it gain or lose time? Why? [HSEB 2072 E]

✎ A simple pendulum having time period 2 seconds is called a second's pendulum. We have,

$$T = 2\pi \sqrt{\frac{l}{g}}$$

And, $T = 2 \text{ sec}$ for a second pendulum on earth.

At moon, value of g decreases but l remains same. So, T increases when it is taken to moon. This means the pendulum loses time when taken to moon.

27. How does the frequency of a simple pendulum related with its length? Hence estimate the frequency of a second's pendulum. [HSEB 2071 C]

✎ The time period T is, $T = 2\pi \sqrt{\frac{l}{g}}$

And, frequency f is, $f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$.

This shows that frequency f of a simple pendulum is inversely proportional to the length of the pendulum. Also, for a second's pendulum,

$T = 2$ secs. So, frequency f of second's pendulum is, $f = \frac{1}{T}$

$$= \frac{1}{2} = 0.5\text{Hz}.$$

28. In usual notation, a simple harmonic motion is given as $y = a \sin(\omega t - \phi)$. Find its acceleration. [HSEB 2071C]

✎ Given, displacement y in SHM is,
 $y = a \sin(\omega t - \phi)$
 The velocity v is,
 $v = \frac{dy}{dt} = \frac{d}{dt} [a \sin(\omega t - \phi)] = a \omega \cos(\omega t - \phi)$.

And, acceleration "a" is,

$$a = \frac{dv}{dt} = \frac{d}{dt} [a \omega \cos(\omega t - \phi)] = a \omega (-\omega) \sin(\omega t - \phi)$$

$$\text{or, } a = -\omega^2 [a \sin(\omega t - \phi)]$$

$$\therefore a = -\omega^2 y$$

29. A SHM is represented as $y = a \cos(\omega t + \phi)$ in usual notation. Find its acceleration. [HSEB 2071 D]

✎ Given, the displacement y in SHM is,
 $y = a \cos(\omega t + \phi)$
 The velocity v is,
 $v = \frac{dy}{dt} = \frac{d}{dt} [a \cos(\omega t + \phi)]$

$$\text{or, } v = -a \omega \sin(\omega t + \phi)$$

And, the acceleration "a" is,
 $a = \frac{dv}{dt} = \frac{d}{dt} [-a \omega \sin(\omega t + \phi)]$

$$= -a \omega \cdot \omega \cos(\omega t + \phi) = -\omega^2 [a \cos(\omega t + \phi)]$$

$$\therefore a = -\omega^2 y$$



Worked Out Examples

1. A body of mass 0.1kg is undergoing simple harmonic motion of amplitude 1m and period 0.2second . if the oscillation is produced by a spring, what will be the maximum value of the force and the force constant of the spring? [NEB 2074]

Solution:

Mass of a body (m) = 0.1 kg
 Amplitude (r) = 1m
 Time period (T) = 0.2m
 Maximum force (F_{\max}) = ?
 Force constant of spring (K) = ?

Since, $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2} = 10\pi \text{ rad/s}$

At extreme position, the acceleration is maximum and it is given by,

$$a_{\max} = \omega^2 r = (10\pi)^2 \times 1 = 100\pi^2 \text{ m/s}^2$$

The force is maximum at extreme position and it is given by,

$$F_{\max} = m a_{\max} = 0.1 \times 100\pi^2 = 10\pi^2 = 98.70\text{N}$$

$$\therefore F_{\max} = 98.70 \text{ N}.$$

We know,

$$T = 2\pi \sqrt{\frac{m}{K}}$$

$$\text{or, } T^2 = 4\pi^2 \frac{m}{K}$$

$$\text{or, } K = 4\pi^2 \frac{m}{T^2} = 4\pi^2 \times \left[\frac{0.1}{(0.2)^2} \right]$$

$$\therefore K = 98.70 \text{ N/m}$$

So, the maximum value of force and force constant of the spring are 98.70N and 98.70 N/m respectively.

2. A small mass of 0.2 kg is attached to one end of helical string and produces an extension of 15mm . The mass is now set into vertical oscillation of amplitude 10mm . What is,
 a. the period of oscillation?
 b. the maximum K.E. of the mass?
 c. the P.E. of the spring when the mass is 5mm below the centre of oscillation? [$g = 9.8\text{ms}^{-2}$] [HSEB 2069 B]

Solution:

Given : $m = 0.2\text{kg}$, $x = 15\text{mm} = 15 \times 10^{-3}\text{m}$, $g = 9.8\text{ms}^{-2}$

Since, $mg = Kx$

$$\text{or, } \frac{m}{K} = \frac{x}{g}$$

$$\text{a. } T = ?$$

We have the relation,

$$T = 2\pi \sqrt{\frac{m}{K}} = 2 \times 3.14 \times \sqrt{\frac{x}{g}}$$

$$= 2 \times 3.14 \times \sqrt{\frac{15 \times 10^{-3}}{9.8}} = 0.25 \text{ sec}$$

$$\text{b. } K.E_{\max} = ?$$

Amplitude (r) = $10\text{mm} = 10 \times 10^{-3}\text{m}$.

We know that,

$$\begin{aligned} K.E_{\max} &= \frac{1}{2} m V_{\max}^2 = \frac{1}{2} m (r\omega)^2 \\ &= \frac{1}{2} m r^2 \omega^2 = \frac{1}{2} m r^2 \left(\frac{2\pi}{T} \right)^2 \\ &= \frac{2\pi^2 m r^2}{T^2} = \frac{2 \times (3.14)^2 \times 0.2 \times (10 \times 10^{-3})^2}{(0.25)^2} \\ &= 6.3 \times 10^{-3} \text{ Joules.} \end{aligned}$$

$$\therefore K.E_{\max} = 6.3 \times 10^{-3} \text{ J}$$

$$c. \text{ P.E.} = ?, \text{ when } x = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$$

We have,

$$\begin{aligned} \text{P.E.} &= \frac{1}{2} m \omega^2 (r^2 - x^2) = \frac{1}{2} m \left(\frac{2\pi}{T} \right)^2 (r^2 - x^2) \\ &= \frac{1}{2} \times 0.2 \times \left(\frac{2 \times 3.14}{0.25} \right)^2 [(10 \times 10^{-3})^2 - (5 \times 10^{-3})^2] \\ &= 4.7 \times 10^{-3} \text{ Joules.} \end{aligned}$$

$$\therefore \text{ P.E.} = 4.7 \times 10^{-3} \text{ J.}$$

3. A simple pendulum has a period of 4.2 second, when the pendulum is shortened by 1m the period is 3.7 second. From these measurements, calculate the acceleration of free fall and the original length of the pendulum. [HSEB 2068]

Solution:

$$\text{Given : } T_1 = 4.2 \text{ sec}$$

$$\text{Let, } l_1 = l$$

When the length is shortened by 1m, then

$$T_2 = 3.7 \text{ secs}$$

$$l_2 = (l - 1)$$

$$g = ? \text{ and } l = ?$$

$$\text{Now, } T_1 = 2\pi \sqrt{\frac{l_1}{g}} \Rightarrow 4.2 = 2\pi \sqrt{\frac{l}{g}} \quad \dots (1)$$

$$\text{and } T_2 = 2\pi \sqrt{\frac{l_2}{g}} \Rightarrow 3.7 = 2\pi \sqrt{\frac{l-1}{g}} \quad \dots (2)$$

Dividing equation (2) by equation (1), we get

$$\frac{3.7}{4.2} = \sqrt{\frac{l-1}{l}}$$

$$\text{Squaring, } \left(\frac{3.7}{4.2} \right)^2 = \frac{l-1}{l}$$

$$\text{or, } \left(\frac{3.7}{4.2} \right)^2 = 1 - \frac{1}{l}$$

$$\text{or, } \frac{1}{l} = 1 - \left(\frac{3.7}{4.2} \right)^2$$

$$\text{or, } \frac{1}{l} = 0.224$$

$$\therefore l = \frac{1}{0.224} = 4.5 \text{ m.}$$

From equation (1),

$$(4.2)^2 = \left(\frac{l}{g} \right) 4\pi^2$$

$$\therefore g = \frac{4\pi^2 l}{(4.2)^2} = \frac{4 \times (3.14)^2 \times 4.5}{(4.2)^2} = 10 \text{ ms}^{-2}$$

4. A second pendulum is taken to moon. If the time period on the surface of the moon is 4.90 seconds, what will be the acceleration due to gravity of the moon? Also, prove that the acceleration due to gravity of the moon to be $\frac{1}{6}$ th of that of the earth.

Solution:

Given : For a second pendulum on the surface of earth,
 $T_1 = 2 \text{ sec}$, $l = 1 \text{ m}$.

When it is taken to the surface of moon, then

$$T_m = 4.90 \text{ seconds.}$$

$$g_m = ?$$

$$l = 1 \text{ m}$$

$$\text{Since, } T_m = 2\pi \sqrt{\frac{l}{g_m}}$$

$$\text{or, } T_m^2 = 4\pi^2 \frac{l}{g_m}$$

$$\text{or, } g_m = 4\pi^2 \frac{l}{T_m^2}$$

$$\text{or, } g_m = 4 \times (3.14)^2 \times \frac{1}{(4.90)^2} = 1.64 \text{ ms}^{-2}$$

$$\text{Now, } \frac{T_E}{T_m} = \frac{2\pi \sqrt{l/g_E}}{2\pi \sqrt{l/g_m}}$$

$$\text{or, } \frac{T_E}{T_m} = \sqrt{\frac{g_m}{g_E}}$$

$$\text{Squaring, } \left(\frac{T_E}{T_m} \right)^2 = \frac{g_m}{g_E}$$

$$\therefore \frac{g_m}{g_E} = \left(\frac{2}{4.90} \right)^2 = 0.167 = \frac{1}{6}$$

5. Calculate the period of revolution of a simple pendulum of length 1.8m with a bob of mass 2.2kg. If the bob of this pendulum is pulled aside a horizontal distance of 20cm and released, what would be the values of (a) the K.E. and (b) the velocity of the bob at the lowest point of the swing? [HSEB 2067]

Solution:

$$\text{Given: } l = 1.8 \text{ m}$$

$$\text{Amplitude (r)} = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$$

$$m = 2.2 \text{ kg} \quad g = 10 \text{ ms}^{-2}$$

$$T = ?,$$

$$\text{The relation, } T = 2\pi \sqrt{\frac{l}{g}} = 2 \times 3.14 \times \sqrt{\frac{1.8}{10}}$$

$$\therefore T = 2.67 \text{ sec}$$

$$a. \text{ K.E. at the lowest point (K.E}_{\max}) = ?$$

Therefore,

$$K.E_{\max} = \frac{1}{2} m V_{\max}^2 = \frac{1}{2} m (r\omega)^2 = \frac{1}{2} m r^2 \omega^2$$

$$= \frac{1}{2} m r^2 \left(\frac{2\pi}{T} \right)^2 = \frac{2\pi^2 m r^2}{T^2}$$

or, $K.E_{\max} = \frac{2 \times (3.14)^2 \times 2.2 \times (20 \times 10^{-2})^2}{(2.67)^2} = 0.24 \text{ J}$

- b. Velocity at the lowest point (v_{\max}) = ?

We know that,

$$v_{\max} = r\omega = r \left(\frac{2\pi}{T} \right)$$

$$= 20 \times 10^{-2} \left(\frac{2 \times 3.14}{2.67} \right) = 0.47 \text{ ms}^{-1}$$

6. The displacement y of a mass vibrating with simple harmonic motion is given by $y = 20\sin 10\pi t$. Where y is in millimeter and t is in second. What is: (a) amplitude, (b) the period and (c) the velocity at $t = 0$. [HSEB 2064]

Solution:

Given, the given equation of displacement is,

$$y = 20\sin 10\pi t \quad \dots (1)$$

Since, the displacement equation of SHM is,

$$y = r \sin \omega t \quad \dots (2)$$

Comparing equations (1) and (2), we get

- a. Amplitude, $r = 20 \text{ mm} = 20 \times 10^{-3} \text{ m}$.

- b. Also, $\omega = 10\pi$

$$\text{or, } \frac{2\pi}{T} = 10\pi$$

$$\text{or, } T = 0.2 \text{ sec}$$

$$\text{c. } v = ? \text{ at } t = 0$$

We have, $v = r\omega \cos \omega t$

At $t = 0$,

$$v = r\omega \cos 0^\circ = r\omega = 20 \times 10^{-3} \times 10\pi = 0.2 \times 3.14 = 0.628 \text{ ms}^{-1}$$

7. A simple pendulum 4m long swings with an amplitude of 0.2m. Compute the velocity of the pendulum at its lowest point and its acceleration at extreme ends. [HSEB 2062, 2050]

Solution:

Given: $l = 4 \text{ m}$ $r = 0.2 \text{ m}$

Velocity at lowest end (v_{\max}) = ?

Acceleration at extreme ends (a_{\max}) = ?

$$\text{Since, } T = 2\pi \sqrt{\frac{l}{g}} = 2 \times 3.14 \times \sqrt{\frac{4}{9.8}} = 4.01 \text{ sec}$$

We know,

$$v_{\max} = r\omega = r \left(\frac{2\pi}{T} \right) = 0.2 \left(\frac{2 \times 3.14}{4.01} \right) = 0.31 \text{ ms}^{-1}$$

Again,

$$a_{\max} = \omega^2 r = \left(\frac{2\pi}{T} \right)^2 r = \left(\frac{2 \times 3.14}{4.01} \right)^2 \times 0.2 = 0.49 \text{ ms}^{-2}$$

8. A particle of mass 0.3kg vibrates with a period of 2 seconds. If its amplitude is 0.5m, what is its maximum kinetic energy? [HSEB 2060]

Solution:

Given: $m = 0.3 \text{ kg}$

$T = 2 \text{ secs}$

$r = 0.5 \text{ m}$

$K.E_{\max} = ?$

Since,

$$K.E_{\max} = \frac{1}{2} m v_{\max}^2 = \frac{1}{2} m (r\omega)^2$$

$$= \frac{1}{2} m r^2 \omega^2 = \frac{1}{2} m r^2 \left(\frac{2\pi}{T} \right)^2 = \frac{2\pi^2 m r^2}{T^2}$$

$$\text{or, } K.E_{\max} = \frac{2 \times (3.14)^2 \times 0.3 \times (0.5)^2}{2^2} = 0.37 \text{ Joules.}$$

9. A small mass rests on a horizontal platform which vibrates in simple harmonic motion with a period of 0.25 second. Find the maximum amplitude of the motion which will allow the mass to remain in contact with the platform throughout the motion. [HSEB 2053]

Solution:

Given: $T = 0.25 \text{ secs}$

Maximum amplitude (r) = ?

For the mass to remain in contact with the platform throughout the motion, the maximum acceleration of the platform should be equal to g , i.e.

$$a_{\max} = g$$

$$\text{or, } \omega^2 r = g$$

$$\text{or, } \left(\frac{2\pi}{T} \right)^2 \times r = g$$

$$\text{or, } r = \left(\frac{T}{2\pi} \right)^2 \times g$$

$$\therefore r = \left(\frac{0.25}{2 \times 3.14} \right)^2 \times 9.8 = 0.0155 \text{ m} = 15.5 \text{ mm}$$

10. A 1.50 kg mass on a spring has displacement as a function of time given by the equation $x(t) = (7.40 \text{ cm}) \cos [(4.16 \text{ s}^{-1})t - 2.42]$. Find (a) the time for one complete vibration; (b) the force constant of the spring; (c) the maximum speed of the mass; and (d) the maximum force on the mass.

Solution:

Given, $m = 1.5 \text{ kg}$,

$$x(t) = 7.4 \text{ cm } [4.16t - 2.42] \quad \dots (1)$$

Comparing equation (1) with the equation of SHM,

$$x = A \cos (\omega t - \phi) \quad \dots (2)$$

We get, $\omega = 4.16 \text{ rad/sec}$

$$A = 7.4 \text{ cm} = 7.4 \times 10^{-2} \text{ m.}$$

- a. $T = ?$

We have,

$$T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{4.16} = 1.51 \text{ sec}$$

- b. $K = ?$

We have,

$$\omega = \sqrt{\frac{K}{m}}$$

$$\text{or, } \omega^2 = \frac{K}{m}$$

$$\text{or, } K = \omega^2 m = (4.16)^2 \times 1.5 = 25.96 \text{ N/m.}$$

c. $v_{\max} = ?$
We have,
 $v = \omega \sqrt{A^2 - x^2}$
For v_{\max} , $x = 0$ (mean position)
 $\therefore v_{\max} = \omega A = 4.16 \times 7.4 \times 10^{-2} = 0.308 \text{ m/s.}$

d. $F_{\max} = ?$
Since,
 $F_{\max} = m a_{\max} = m(\omega^2 A) = (m \omega^2) A$
 $= KA = 25.96 \times 7.4 \times 10^{-2} = 1.92 \text{ N}$

11. A 42.5 kg chair is attached to a spring and allowed to oscillate. When the chair is empty, the chair takes 1.30 sec to make one complete vibration. But with a person sitting in it, with her feet off the floor, the chair now takes 2.54 sec for one cycle. What is the mass of the person?

Solution:

Given : Mass of the chair (m_1) = 42.5 kg

$T_1 = 1.3$ secs.

Mass of the person = m (say).

So, total mass (m_2) = $m_1 + m$

$T_2 = 2.54$ sec

$$\text{Time period } T = 2\pi \sqrt{\frac{m}{K}}$$

$$\therefore T \propto \sqrt{m}$$

$$\text{Therefore, } \frac{T_2}{T_1} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{m_1 + m}{m_1}} = \sqrt{1 + \frac{m}{m_1}}$$

$$\text{or, } \left(\frac{T_2}{T_1}\right)^2 = 1 + \frac{m}{m_1}$$

$$\text{or, } m = \left[\left(\frac{T_2}{T_1}\right)^2 - 1\right] m_1$$

$$\therefore m = \left[\left(\frac{2.54}{1.3}\right)^2 - 1\right] 42.5 = 119.75 \text{ kg}$$

So, the mass of the person is 119.75 kg.

12. A 5.00 kg partridge is suspended from a pear tree by an ideal spring of negligible mass. When the partridge is pulled down 0.100 m below its equilibrium position and released, it vibrates with a period of 4.20 sec. (a) What is its speed as it passes through the equilibrium position? (b) What is its acceleration when it is 0.050 m above the equilibrium position? (c) When it is moving upward, how much time is required for it to move from a point 0.050 m below its equilibrium position to a point 0.050 m above it?

Solution:

Given : $m = 5$ kg; $r = 0.1$ m $T = 4.2$ sec

a. At equilibrium position,
the speed is maximum i.e. $v_{\max} = ?$

We have,

$$v_{\max} = r\omega = r \left(\frac{2\pi}{T}\right) = 0.1 \left(\frac{2 \times 3.14}{4.2}\right) = 0.15 \text{ m/s.}$$

b. $a = ?$ when $x = 0.05$ m

We have,

$$a = -\omega^2 x = \left(\frac{2\pi}{T}\right)^2 x$$

$$= - \left(\frac{2 \times 3.14}{4.2}\right)^2 \times 0.05 = -0.112 \text{ m/s}^2.$$

The negative sign means the downward acceleration.

c. $t = ?$ from $x = -0.05$ m to $x = 0.05$ m.

Time from $x = -0.05$ m to equilibrium position

($x = 0$) i.e. for $x = \frac{r}{2}$, can be calculated as;

$$x = r \sin \omega t$$

$$\text{or, } \frac{r}{2} = r \sin \omega t$$

$$\text{or, } \sin \omega t = \frac{1}{2}$$

$$\text{or, } \omega t = \sin^{-1} \left(\frac{1}{2}\right)$$

$$\text{or, } \left(\frac{2\pi}{T}\right) \cdot t = \frac{\pi}{6}$$

$$\text{or, } \left(\frac{2}{4.2}\right) \cdot t = \frac{1}{6}$$

$$\therefore t = \frac{4.2}{6 \times 2} = 0.35 \text{ sec}$$

Again, time take to move from equilibrium position

($x = 0$) to $x = 0.05$ m above is 0.35 sec. So, total time taken is $= 2t = 2 \times 0.35 = 0.70$ sec

13. After landing on an unfamiliar planet, a space explorer constructs a simple pendulum of length 50 cm. She finds that the pendulum makes 100 complete swings in 136 seconds. What is the value of g on this planet? [HSEB 2068, S]

Solution:

Given:

$$l = 50 \text{ cm} = 0.5 \text{ m}$$

$$\text{frequency } (f) = \frac{100}{136} \text{ Hz}$$

$g = ?$

$$\text{Since, } T = \frac{1}{f} = \frac{136}{100} = 1.36 \text{ sec.}$$

$$\text{Also, } T = 2\pi \sqrt{\frac{l}{g}}$$

$$\text{or, } T^2 = 4\pi^2 \frac{l}{g}$$

$$\therefore g = 4\pi^2 \frac{l}{T^2} = \frac{4 \times (3.14)^2 \times 0.5}{(1.36)^2} = 10.67 \text{ ms}^{-2}$$

14. A certain simple pendulum has a period on the earth of 1.60 sec. What is its period on the surface of mars, where $g = 3.71 \text{ ms}^{-2}$.

Solution:

Given: On Earth

$$T_E = 1.60 \text{ sec}$$

$$g_E = 10 \text{ ms}^{-2}$$

On Mars

$$T_m = ?$$

$$g_m = 3.71 \text{ ms}^{-2}$$

$$\text{Since, } T_E = 2\pi \sqrt{\frac{l}{g}}$$

$$\text{or, } T_E^2 = 4\pi^2 \frac{l}{g_E}$$

$$\text{or, } l = \frac{T_E^2 \times g_E}{4\pi^2}$$

$$\text{or, } l = \frac{(1.60)^2 \times 10}{4 \times (3.14)^2}$$

$$\therefore l = 0.65 \text{ m}$$

$$\text{Now, } T_m = 2\pi \sqrt{\frac{l}{g_m}} = 2 \times 3.14 \times \sqrt{\frac{0.65}{3.71}} = 2.63 \text{ sec.}$$

15. An object moves in SHM with a period of 0.500 sec. The object's maximum acceleration is 6.4 m/s^2 . What is its maximum speed?

Solution:

Given : Period (T) = 0.500 sec

$$a_{\max} = 6.4 \text{ m/s}^2$$

$$v_{\max} = ?$$

$$\text{Since, } a_{\max} = \omega^2 r$$

$$\text{or, } a_{\max} = \left(\frac{2\pi}{T}\right)^2 r$$

$$\text{or, } r = a_{\max} \times \left(\frac{T}{2\pi}\right)^2 = 6.4 \times \left(\frac{0.5}{2 \times 3.14}\right)^2 = 0.041 \text{ m}$$

$$\text{Now, } v_{\max} = r\omega = r \left(\frac{2\pi}{T}\right)$$

$$= 0.041 \times \left(\frac{2 \times 3.14}{0.5}\right) = 0.51 \text{ m/s}^2.$$

16. On the planet Newtonian, a simple pendulum having a bob with mass 1.25 kg and a length 185 cm takes 1.42 sec, when released from rest to swing through an angle of 12.5° , where it again has zero speed. The circumference of Newtonian is measured to be 51,400 km. What is the mass of the planet Newtonian?

Solution:

$$\text{Given: } m = 1.25 \text{ kg} \quad l = 185 \text{ cm} = 1.85 \text{ m}$$

$$\text{Time for half oscillation} = 1.42 \text{ sec}$$

$$\text{Time to complete one oscillation is,}$$

$$T = 2 \times 1.42 = 2.84 \text{ sec}$$

$$\text{Mass of the planet Newtonian (M) = ?}$$

$$\text{Since, } T = 2\pi \sqrt{\frac{l}{g}}$$

$$\text{or, } T^2 = 4\pi^2 \left(\frac{l}{g}\right), \text{ squaring}$$

$$\text{or, } g = 4\pi^2 \left(\frac{l}{T^2}\right)$$

$$\text{or, } g = 4 \times (3.14)^2 \times \left[\frac{1.85}{(2.84)^2}\right]$$

$$\therefore g = 9.05 \text{ ms}^{-2}.$$

$$\text{Now, circumference of the planet,}$$

$$C = 51,400 \text{ Km} = 5.14 \times 10^7 \text{ m.}$$

$$\text{Since,}$$

$$\text{or, } C = 2\pi R$$

$$\text{or, } R = \frac{C}{2\pi} = \frac{5.14 \times 10^7}{2 \times 3.14} = 8.18 \times 10^6 \text{ m.}$$

$$\text{Again, we know,}$$

$$g = \frac{GM}{R^2}$$

$$\text{or, } M = \frac{gR^2}{G} = \frac{9.05 \times (8.18 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$\therefore M = 9.08 \times 10^{24} \text{ kg.}$$



Additional Numerical Examples

1. A machine part is undergoing S.H.M with a frequency of 5.00 Hz and amplitude 1.80 cm. How long does it take the part to go from $x = 0$ to $x = -1.80 \text{ cm}$?

Solution:

$$\text{Given } f = 5 \text{ Hz}$$

$$\text{Amplitude (r) = 1.8 cm}$$

$$\text{time (t) = ? for } x = 0 \text{ to } x = -1.8 \text{ cm}$$

$$\text{Here, } 0 \text{ to } -1.8 \text{ cm is the displacement from mean position}$$

$$\text{to one end of the extreme position which is } \frac{1}{4}^{\text{th}} \text{ of time}$$

$$\text{period the total distance for one complete oscillation.}$$

So, time taken

$$t = \frac{1}{4} \times (T) = \frac{1}{4} \times \left(\frac{1}{f}\right) = \frac{1}{4} \times \frac{1}{5} = \frac{1}{20} = 0.05 \text{ sec}$$

2. The tip of a tuning fork goes through 440 complete vibration in 0.500 seconds. Find the angular frequency and the period of the motion.

Solution:

$$\text{Given : No. of vibrations (n) = 440 } t = 0.500 \text{ secs.}$$

$$\text{Since, time period (T) is the time for one vibration.}$$

$$\therefore T = \frac{t}{n} = \frac{0.500}{440} = 1.14 \times 10^{-3} \text{ sec}$$

And the angular frequency,

$$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{1.14 \times 10^{-3}} = 5.53 \times 10^3 \text{ rad/sec.}$$

3. A 2.00 kg frictionless block is attached to an ideal spring with force constant 300 N/m. At $t = 0$, the spring is neither stretched nor compressed and the block is moving in the negative direction at 12.0 m/s. Find (a) the amplitude and (b) the phase angle.

Solution:

Given: $m = 2 \text{ kg}$ $K = 300 \text{ N/m}$
 $v_{\max} = 12 \text{ m/s}$ (at mean position)

- a. Amplitude (r) = ?

$$\text{Since, } v = \omega \sqrt{r^2 - x^2}$$

At mean position, $x = 0$ and $v = v_{\max}$

$$\therefore v_{\max} = \omega r$$

$$\text{or, } r = \frac{v_{\max}}{\omega} = \frac{12}{\sqrt{K/m}} = \frac{12}{\sqrt{300/2}} = \frac{12}{\sqrt{150}} = 0.98 \text{ m}$$

- b. $\phi = ?$

$$\text{Since, } x = r \cos(\omega t + \phi)$$

At $t = 0$, $x = 0$

$$\therefore 0 = r \cos \phi$$

$$\text{or, } \cos \phi = 0$$

$$\therefore \phi = \frac{\pi}{2}$$

4. The point of a sewing machine moves in SHM along the x -axis with a frequency of 2.5 Hz. At $t = 0$ its position component is +1.1 cm. Find the acceleration component of the needle at $t = 0$.

Solution:

Given:

$$f = 2.5 \text{ Hz} \quad x = 1.1 \text{ cm} = 1.1 \times 10^{-2} \text{ m at } t = 0.$$

$a = ?$

We have,

$$a = \omega^2 x = (2\pi f)^2 x = (2 \times 3.14 \times 2.5)^2 \times 1.1 \times 10^{-2} = 2.71 \text{ m/s}^2$$

5. An object is undergoing SHM with period 1.200 sec and amplitude 0.600 m. At $t = 0$, the object is at $x = 0$. How far is the object from the equilibrium position when $t = 0.480$ sec?

Solution :

Given, $T = 1.2 \text{ sec}$ $r = 0.6 \text{ m}$

$x = 0$ at $t = 0$ $x = ?$ at $t = 0.48 \text{ sec}$

We have,

$$x = r \sin \omega t = r \sin \left(\frac{2\pi}{T} t \right)$$

$$= 0.6 \sin \left[\frac{2 \times 3.14}{1.2} \times 0.48 \right] = 0.026 \text{ m}$$

6. A 0.400 kg object undergoing SHM has $a_x = -2.70 \text{ m/s}^2$, when $x = 0.300 \text{ m}$. What is the time for one oscillation?

Solution :

Given, $m = 0.4 \text{ kg}$, $a_x = -2.7 \text{ m/s}^2$ when $x = 0.3 \text{ m}$,
 $T = ?$

We have the relation,

$$F = ma_x = -Kx$$

$$\therefore \frac{m}{K} = -\frac{x}{a_x} = \frac{0.3}{(-2.7)} = \frac{1}{9}$$

$$\text{Since, } T = 2\pi \sqrt{\frac{m}{K}} = 2 \times 3.14 \times \sqrt{\frac{1}{9}} = 2.09 \text{ sec}$$

7. A 0.500 kg mass on a spring has velocity as a function of time given by $v_x(t) = (3.60 \text{ cm/s}) \sin[(4.71 \text{ s}^{-1})t - \frac{\pi}{2}]$. What is (a) the period ? (b) the amplitude ? and (c) the maximum acceleration of the mass?

Solution:

$$\text{Given: } v_x(t) = (3.6) \sin [(4.71)t - \frac{\pi}{2}] \dots (1)$$

We know, the equation of SHM is,

$$v_x = -r\omega \sin(\omega t - \phi) \dots (2)$$

Comparing equations (1) and (2), we get

$$-r\omega = 3.6$$

$$\omega = 4.71$$

$$\phi = \frac{\pi}{2}$$

- a. $T = ?$

$$\text{Since, } T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{4.71} = 1.33 \text{ sec}$$

- b. $r = ?$

$$\text{Since, } r\omega = 3.6 \quad (\text{taking magnitude only})$$

$$\therefore r = \frac{3.6}{\omega} = \frac{3.6}{4.71} = 0.764 \text{ cm} = 0.764 \times 10^{-2} \text{ m}$$

- c. $a_{\max} = ?$

$$a_{\max} = \omega^2 r = (4.71)^2 \times 0.764 \times 10^{-2} = 16.96 \times 10^{-2} \text{ m/s}^2$$

8. You pull a simple pendulum of length 0.240 m to the side through an angle of 3.50° and release it. (a) How much time does it take pendulum bob to reach its highest speed? (b) How much time does it take if the pendulum is released at an angle of 1.75° instead of 3.50° ?

Solution:

Given: $l = 0.24 \text{ m}$

- a. $\theta = 3.50^\circ$

Here $\theta < 4^\circ$, the motion is SHM and,

$$T = 2\pi \sqrt{\frac{l}{g}} \dots (1)$$

The bob has the maximum speed at the bottom (mean position).

$$\begin{aligned}\text{So, time taken (t)} &= \frac{T}{4} = \frac{1}{4} 2\pi \sqrt{\frac{l}{g}} = \frac{\pi}{2} \sqrt{\frac{l}{g}} \\ &= \frac{3.14}{2} \sqrt{\frac{0.24}{9.8}} = 0.25 \text{ sec}\end{aligned}$$

- b. For $\theta < 4^\circ$, the time period (T) is independent with the amplitude of oscillation. So, time taken is same.
i.e. $t = \frac{T}{4} = 0.25 \text{ sec}$

9. The scale of a spring balance reading from zero to 200 N is 12.5 cm long. A fish hanging from the bottom of the spring oscillates vertically at 2.60 Hz. What is the mass of the fish? You can ignore the mass of the spring.

Solution:

Given, When $F = 200 \text{ N}$, then $x = 12.5 \text{ cm} = 12.5 \times 10^{-2} \text{ m}$

$F = 2.60 \text{ Hz}$

$m = ?$

Since, $F = Kx$

$$\text{or, } K = \frac{F}{x} = \frac{200}{12.5 \times 10^{-2}} = 1600 \text{ N/m.}$$

$$\text{Also, } f = \frac{1}{2\pi} \sqrt{\frac{K}{m}}$$

$$\text{or, } f^2 = \frac{1}{4\pi^2} \cdot \frac{K}{m}$$

$$\therefore m = \frac{K}{4\pi^2 f^2} = \frac{1600}{4 \times (3.14)^2 \times (2.6)^2} = 6 \text{ kg}$$

10. A 0.500 kg glider, attached to the end of an ideal spring with force constant $k = 450 \text{ N/m}$, undergoes SHM with an amplitude 0.040 m. Compute, (a) the maximum speed of the glider, (b) the speed of the glider when it is at $x = -0.015 \text{ m}$.

Solution:

Given: $m = 0.5 \text{ kg}$

$K = 450 \text{ N/m}$

$r = 0.04 \text{ m}$

a. $v_{\max} = ?$

We have,

$$v_{\max} = r\omega = r \sqrt{\frac{K}{m}} = 0.04 \times \sqrt{\frac{450}{0.5}} = 1.2 \text{ ms}^{-1}$$

b. $v = ?$ when $x = 0.015 \text{ m}$.

We have,

$$\begin{aligned}v &= \omega \sqrt{r^2 - x^2} = \sqrt{\frac{K}{m}} \times \sqrt{r^2 - x^2} \\ &= \sqrt{\frac{450}{0.5}} \times \sqrt{(0.04)^2 - (0.015)^2} = 30 \times 0.037\end{aligned}$$

$$\therefore v = 1.11 \text{ m/s}$$

11. An object moving with simple harmonic motion has an amplitude of 0.02 m and a frequency of 20 Hz. Calculate (a) the period of oscillation, (b) the acceleration at the middle and end of an oscillation, and (c) velocities at the corresponding instants.

Solution:

Given, $r = 0.02 \text{ m}$ $f = 20 \text{ Hz}$

a. $T = ?$

$$\text{Since, } T = \frac{1}{f} = \frac{1}{20} = 0.5 \text{ secs.}$$

b. Acceleration at mean position (a_{\min}) = ?

$$\text{Since, } a_{\min} = \omega^2 y = \omega^2 \times 0 = 0$$

Acceleration at extreme position (a_{\max}) = ?

$$\begin{aligned}a_{\max} &= \omega^2 r = (2\pi f)^2 r = (2 \times \pi \times 20)^2 \times 0.02 \\ &= 32\pi^2 \text{ m/s}^2 = 315.8 \text{ ms}^{-2}\end{aligned}$$

c. At mean position (v_{\max}) = ?

At extreme position (v_{\min}) = ?

$$\text{We have, } v = \omega \sqrt{r^2 - y^2}$$

At mean position, $y = 0 \Rightarrow v_{\max} = r\omega$

$$\text{or, } v_{\max} = r(2\pi f) = 0.02(2 \times \pi \times 20) = 0.8\pi \text{ ms}^{-1}$$

At extreme position, $y = r$

$$\therefore v_{\min} = \omega \sqrt{r^2 - r^2} = 0$$

12. A steel strip, clamped at one end, vibrates with a frequency of 50 Hz and an amplitude of 8 mm at the free end. Find (a) the velocity of the end when passing through the zero position, (b) the acceleration at the maximum displacement.

Solution:

Given: $f = 50 \text{ Hz}$

$r = 8 \text{ mm} = 8 \times 10^{-3} \text{ m}$

a. Velocity at mean position, $v_{\max} = ?$

$$\text{Since, } v_{\max} = r\omega = r \times 2\pi f = 8 \times 10^{-3} \times 2\pi \times 50$$

$$\therefore v_{\max} = 0.8\pi \text{ ms}^{-1} = 2.51 \text{ ms}^{-1}$$

b. Acceleration at the extreme position, $a_{\max} = ?$

$$\text{Since, } a_{\max} = r\omega^2 = r \times (2\pi f)^2$$

$$= 8 \times 10^{-3} \times (2\pi \times 50)^2$$

$$= 80\pi^2 \text{ ms}^{-2} = 789.57 \text{ ms}^{-2}$$

$$\therefore a_{\max} \approx 790 \text{ ms}^{-2}$$

13. A spring of force constant K of 5 N m^{-1} ($F = -KX$) is placed horizontally on a smooth table. One end of the spring is fixed and a mass X of 0.20 kg is attached to the free end. X is displaced a distance of 4 mm along the table and then released. Show that the motion of X is simple harmonic and calculate (a) the period, (b) the maximum acceleration, (c) the maximum kinetic energy and (d) the maximum potential energy of the spring.

Solution:

Given: Force constant (K) = 5 N m^{-1}

Mass (m) = 0.20 kg

Amplitude (r) = 4 mm = $4 \times 10^{-3} \text{ m}$

a. $T = ?$

$$\text{Since, } T = 2\pi \sqrt{\frac{m}{K}} = 2 \times 3.14 \times \sqrt{\frac{0.2}{5}} = 1.26 \text{ sec}$$

b. $a_{\max} = ?$

$$\begin{aligned}\text{Since, } a_{\max} &= \omega^2 r = \left(\frac{2\pi}{T}\right)^2 r \\ &= \left(\frac{2 \times 3.14}{1.26}\right)^2 \times 4 \times 10^{-3} = 0.1 \text{ ms}^{-2}.\end{aligned}$$

c. $K.E_{\max} = ?$

$$\begin{aligned}\text{Since, } K.E_{\max} &= \frac{1}{2} m v_{\max}^2 = \frac{1}{2} m (r\omega)^2 \\ &= \frac{1}{2} m r^2 \omega^2 = \frac{1}{2} m r^2 \left(\frac{2\pi}{T}\right)^2 \\ \therefore K.E_{\max} &= \frac{2\pi^2 m r^2}{T^2} = \frac{2 \times (3.14)^2 \times 0.2 \times (4 \times 10^{-3})^2}{(1.26)^2} \\ &= 3.98 \times 10^{-5} \text{ Joules}.\end{aligned}$$

d. $P.E_{\text{mzx}} = ?$

$$P.E_{\max} = K.E_{\max} = 3.98 \times 10^{-5} \text{ Joules}.$$

14. A mass X of 0.1 kg is attached to the free end of a vertical helical spring whose upper end is fixed and the spring extends by 0.04 m . X is now pulled down a small distance 0.02 m and then released. Find (a) its period, (b) the maximum force acting on it during the oscillations and (c) its kinetic energy when X passes through its mean position [$g = 10 \text{ ms}^{-2}$]

Solution:

Given:

$$\begin{aligned}m &= 0.1 \text{ kg} & x &= 0.04 \text{ m} \\ r &= 0.02 \text{ m} & g &= 10 \text{ ms}^{-2}\end{aligned}$$

a. $T = ?$

We have the relation,

$$Kx = mg$$

$$\text{or, } \frac{m}{K} = \frac{x}{g}.$$

$$\begin{aligned}\text{Now, } T &= 2\pi \sqrt{\frac{m}{K}} = 2 \times 3.14 \times \sqrt{\frac{x}{g}} \\ &= 2 \times 3.14 \times \sqrt{\frac{0.04}{10}} = 0.4 \text{ sec}\end{aligned}$$

b. $F_{\max} = ?$

$$\begin{aligned}F_{\max} &= m a_{\max} = m(\omega^2 r) = m r \omega^2 = m r \left(\frac{2\pi}{T}\right)^2 \\ &= 0.1 \times 0.02 \times \left(\frac{2 \times 3.14}{0.4}\right)^2 = 0.5 \text{ N}\end{aligned}$$

c. At mean position, K.E. is maximum

$$\begin{aligned}\therefore K.E_{\max} &= \frac{1}{2} m v_{\max}^2 = \frac{1}{2} m (r\omega)^2 \\ \therefore K.E_{\max} &= \frac{1}{2} m r^2 \omega^2 = \frac{1}{2} m r^2 \left(\frac{2\pi}{T}\right)^2 = \frac{2\pi^2 m r^2}{T^2} \\ &= \frac{2 \times (3.14)^2 \times 0.1 \times (0.02)^2}{(0.4)^2} \\ &= 5.0 \times 10^{-3} \text{ Joules}\end{aligned}$$

15. When a metal cylinder of mass 0.2 kg is attached to lower end of a light helical spring the upper end of which is fixed, the spring extends by 0.16 m . The metal cylinder is then pulled down a further 0.08 m . (i) Find the force that must be exerted to keep it there if Hooke's law is obeyed, (ii) The cylinder is then released. Find the period of vertical oscillations, and the kinetic energy the cylinder possess when it passes through its mean position. [$g = 10 \text{ ms}^{-2}$]

Solution:

$$\begin{aligned}\text{Given: } m &= 0.2 \text{ kg} & x &= 0.16 \text{ m} \\ r &= 0.08 \text{ m} & g &= 10 \text{ ms}^{-2}\end{aligned}$$

a. $F_{\max} = ?$

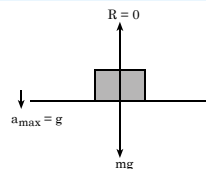
$$\text{Since, } mg = Kx$$

$$\text{or, } \frac{m}{K} = \frac{x}{g}$$

$$\begin{aligned}\text{Now, } T &= 2\pi \sqrt{\frac{m}{K}} \\ &= 2\pi \sqrt{\frac{x}{g}} \\ &= 2 \times 3.14 \times \sqrt{\frac{0.16}{10}} = 0.8 \text{ secs}.\end{aligned}$$

$$\begin{aligned}\text{Therefore, } F_{\max} &= m a_{\max} = m(\omega^2 r) = m r \left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2 m r}{T^2} \\ &= \frac{4 \times (3.14)^2 \times 0.2 \times 0.08}{(0.8)^2} = 0.04 \text{ Joules}.\end{aligned}$$

16. A particle rests on a horizontal platform which is moving vertically in simple harmonic motion with an amplitude of 10 cm . Above a certain frequency, the thrust between the particle and the platform would become zero at some point in the motion. What is this frequency, and at what point in the motion does the thrust become zero at this frequency?



Solution:

$$\text{Given, } r = 10 \text{ cm} = 10 \times 10^{-2} \text{ m, } f = ?$$

At the top of oscillation, the horizontal platform is moving downward with $a_{\max} = g$ and hence the reaction thrust between the particle and the platform would become zero.

$$\text{Since, } a_{\max} = g$$

$$\text{or, } r\omega^2 = g$$

$$\text{or, } r(2\pi f)^2 = g$$

$$\text{or, } 4\pi^2 r f^2 = g$$

$$\therefore f = \frac{1}{2\pi} \sqrt{\frac{g}{r}} = \frac{1}{2 \times 3.14} \sqrt{\frac{10}{10 \times 10^{-2}}} = 1.6 \text{ Hz}.$$

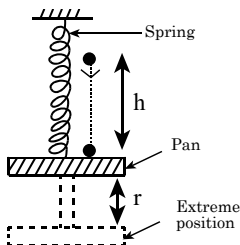
17. A helical spring gives a displacement of 5 cm for a load of 500 gm. Find the maximum displacement produced when a mass of 80 gm is dropped from a height of 10 cm on to a light pan attached to the spring.

Solution:

Given: $x = 5 \text{ cm} = 0.05 \text{ m}$ $m = 500 \text{ gm} = 0.5 \text{ kg}$

Since, $Kx = mg$

$$\text{or, } K = \frac{mg}{x} = \frac{0.5 \times 10}{0.05} = 100 \text{ Nm}^{-1}.$$



Second part:

Now, we assume that this mass 500gm is not in the pan and the mass of the string and pan are negligible.

Here, $m = 80 \text{ gm} = 0.08 \text{ kg}$

$h = 10 \text{ cm} = 0.1 \text{ m}$, $g = 10 \text{ ms}^{-2}$

Maximum displacement (r) = ?

At extreme position, P.E. of the spring = $\frac{1}{2} Kr^2$

Loss of P.E. of the mass = $mgh + mgr$.

From energy conservation principle,

$$\frac{1}{2} Kr^2 = mgh + mgr$$

$$\text{or, } \frac{1}{2} \times 100 \times r^2 = 0.08 \times 10 \times 0.1 + 0.08 \times 10 \times r$$

$$\text{or, } 50r^2 - 0.8r - 0.08 = 0$$

$$\text{or } r = \frac{-(-0.8) \pm \sqrt{(-0.8)^2 - 4 \times 50 \times (-0.08)}}{2 \times 50}$$

$$\text{or, } r = \frac{0.8 \pm 4.08}{100}$$

$$\text{or, } r = \frac{4.88}{100} \text{ [since, } r \text{ can't be negative]}$$

$$\therefore r = 4.88 \times 10^{-2} \text{ m} \approx 5 \times 10^{-2} \text{ m}$$

18. A simple pendulum of length 1.5 m has a bob of mass 2.0 kg (a) state the formula for the period of small oscillation and evaluate it in this case. (b) If with the string taut, the bob is pulled aside a horizontal distance of 0.15 m from the mean position and then released from rest, find the kinetic energy and the speed with which it passes through the mean position (c) After 50 complete swings, the maximum horizontal displacement of the bob has become only 0.10 m. What fraction of the initial energy has been lost? (d) Estimate the maximum horizontal displacement of the bob after a further 50 complete swings. [Take g to be 10 ms^{-2}].

Solution:

Given: $l = 1.5 \text{ m}$ $m = 2 \text{ kg}$

$$\text{a. } T = 2\pi \sqrt{\frac{l}{g}} = 2 \times 3.14 \times \sqrt{\frac{1.5}{10}} = 2.43 \text{ secs.}$$

$$\text{b. } r_1 = 0.15 \text{ m}$$

At mean position, $v_{\text{max}} = ?$ $K.E_{\text{max}} = ?$

$$v_{\text{max}} = r_1 \omega = r_1 \times \frac{2\pi}{T} = 0.15 \times \frac{2 \times 3.14}{2.43} = 0.387 \text{ m/s}$$

$$\text{And, } K.E_{\text{max}} = \frac{1}{2} mv_{\text{max}}^2 = \frac{1}{2} \times 2 \times (0.387)^2 = 0.15 \text{ Joules.}$$

$$\therefore E_{1(\text{max})} = 0.15 \text{ Joules}$$

c. After 50 swings,

$$r_2 = 0.10 \text{ m, } E_{2(\text{max})} = ?$$

We have,

$$E_{2(\text{max})} = \frac{1}{2} m v_{2(\text{max})}^2 = \frac{1}{2} m (r_2 \omega)^2$$

$$= \frac{1}{2} m r_2^2 \omega^2 = \frac{1}{2} m r_2^2 \left(\frac{2\pi}{T} \right)^2$$

$$\therefore E_{2(\text{max})} = \frac{2\pi^2 m r_2^2}{T^2} = \frac{2 \times (3.14)^2 \times 2 \times (0.10)^2}{(2.43)^2}$$

$$= 0.067 \text{ Joules}$$

$$\text{Fraction of energy lost} = \frac{E_{1(\text{max})} - E_{2(\text{max})}}{E_{1(\text{max})}}$$

$$= \frac{0.15 - 0.067}{0.15} = 0.56$$

d. After next 50 swings, $r_3 = ?$

Let $E_{3(\text{max})}$ be the maximum K.E. after 50 complete swings. Assuming the energy is lost in same fraction, we have

$$\frac{E_{1(\text{max})} - E_{2(\text{max})}}{E_{1(\text{max})}} = \frac{E_{2(\text{max})} - E_{3(\text{max})}}{E_{2(\text{max})}}$$

$$\text{or, } 0.56 = 1 - \frac{E_{3(\text{max})}}{E_{2(\text{max})}}$$

$$\text{or, } \frac{E_{3(\text{max})}}{E_{2(\text{max})}} = 1 - 0.56$$

$$\frac{1}{2} m v_{3(\text{max})}^2 = 0.44$$

$$\text{or, } \frac{1}{2} m v_{2(\text{max})}^2 = 0.44$$

$$\text{or, } \frac{(r_3 \omega)^2}{(r_2 \omega)^2} = 0.44$$

$$\text{or, } r_3^2 = 0.44 \times r_2^2 = 0.44 \times (0.10)^2$$

$$\text{or, } r_3^2 = 4.4 \times 10^{-3}$$

$$\therefore r_3 = \sqrt{4.4 \times 10^{-3}} = 0.067 \text{ m}$$

Hence, maximum horizontal displacement of the bob is 0.067 m.

19. If $M = 0.30 \text{ kg}$, $Kk = 30 \text{ Nm}^{-1}$ and the initial displacement of the mass is 0.015 m, calculate

a. the maximum kinetic energy of the mass,

b. the maximum and minimum values of the tension in the string during the motion.

Solution:

 Given: $M = 0.30 \text{ kg}$ $K = 30 \text{ Nm}^{-1}$
 $r = 0.015 \text{ m}$

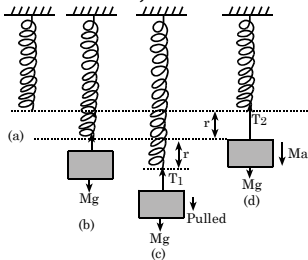
 a. $\text{K.E.}_{\text{max}} = ?$

$$\text{Since, } T = 2\pi\sqrt{\frac{M}{K}} = 2 \times 3.14 \times \sqrt{\frac{0.3}{30}} = 0.63 \text{ secs.}$$

$$\text{Now, } \text{K.E.}_{\text{max}} = \frac{1}{2} M v_{\text{max}}^2 = \frac{1}{2} M (\omega r)^2 = \frac{1}{2} M r^2 \left(\frac{2\pi}{T} \right)^2$$

$$\therefore \text{K.E.}_{\text{max}} = \frac{2\pi^2 M r^2}{T^2} = \frac{2 \times (3.14)^2 \times 0.30 \times (0.015)^2}{(0.63)^2}$$

$$\therefore \text{K.E.}_{\text{max}} = 3.4 \times 10^{-3} \text{ Joules.}$$


 b. Maximum tension (T_{max}) = ?

 Minimum tension (T_{min}) = ?

Here, Fig (a) is the unstretched position of string.

Fig(b) is stretched string when load M is suspended.

Fig (c) is the lowest extreme position of the string

when the load is pulled down.

Here, the resultant upward force is,

$$Ma = T_1 - Mg$$

$$\therefore T_1 = M(g + a) \quad \dots (1)$$

Figure (d) is the upper extreme position of the string when the load is oscillating in SHM.

So, the resultant downward force is,

$$Ma = mg - T_2$$

$$\therefore T_2 = M(g - a) \quad \dots (2)$$

 From (1) and (2), it is clear that the tension on the string is maximum at the lowest position i.e. T_1 and minimum at the upper extreme position i.e. T_2 .

We know, at the extreme position, the acceleration is maximum.

$$\therefore a = \omega^2 r = \left(\frac{2\pi}{T} \right)^2 r = \left(\frac{2 \times 3.14}{0.63} \right)^2 \times 0.015$$

$$\text{or, } a = 1.5 \text{ ms}^{-2}$$

From (1), maximum tension is,

$$T_{\text{max}} = T_1 = M(g + a) = 0.30 (10 + 1.5) = 3.45 \text{ N (at the lower extreme position)}$$

From (2), the minimum tension is,

$$T_{\text{min}} = T_2 = M(g - a) = 0.30 (10 - 1.5) = 2.55 \text{ N (at the upper extreme position)}$$


Exercise
A. Multiple Choice Questions

Circle the best alternative to the following questions:

- The time period of a simple pendulum is double when
 - its length is doubled
 - mass of bob is doubled
 - the length is made four times
 - the mass of bob and length of pendulum are doubled.
- A pendulum suspended from the ceiling of a train has a period T when the train is at rest. When the train accelerates with a uniform acceleration a , the period of oscillation will
 - increases
 - decreases
 - becomes unaffected
 - becomes infinite
- The vertical extension in a light spring by a weight of 1 kg suspended from the wire is 9.8 cm . The period of oscillation is
 - $20\pi \text{ sec}$
 - $2\pi \text{ sec}$
 - $\frac{2\pi}{10} \text{ sec}$
 - $200\pi \text{ sec}$
- A particle is vibrating simple harmonically with an amplitude ' a '. The displacement of the particle when its

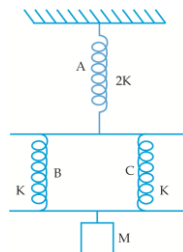
energy is half kinetic and half potential is

- $a/2$
 - $a/\sqrt{2}$
 - $a/4$
 - zero
- A simple pendulum with a bob mass m swing with an angular displacement of 40° . When its angular displacement is 20° , the tension in the string is
 - $mg \cos 20^\circ$
 - greater than $mg \cos 20^\circ$
 - less than $mg \cos 20^\circ$
 - $mg \cos 40^\circ$
 - The maximum velocity and maximum acceleration of a particle executing SHM are 4 m/s & 2 m/s^2 . Then its time period of oscillation will be
 - $\pi/2 \text{ sec}$
 - $2\pi \text{ sec}$
 - $2/\pi \text{ sec}$
 - $4\pi \text{ sec}$
 - A body is executing SHM with amplitude A_0 . At what displacement from mean position its KE will be half its maximum value?
 - $A_0/2$
 - $A_0/4$
 - $\sqrt{3}/2 A_0$
 - $A_0/\sqrt{2}$

8. A body is executing SHM with amplitude A_0 . At what displacement from mean position its velocity is half its maximum value?
- $A_0/2$
 - $\sqrt{3}/2 A_0$
 - $A_0/\sqrt{2}$
 - $A_0/4$
9. Starting from mean position, a particle in SHM takes time t_1 and t_2 to cover first half and next half displacement in moving from mean to extreme position. Then.
- $t_1 = t_2$
 - $t_1 = 2t_2$
 - $t_1 = 1/2 t_2$
 - $t_1 = t_2/\sqrt{2}$
10. Two simple harmonic motion are given by $x_1 = a \sin \omega t$ and $x_2 = b \cos \omega t$. The phase difference between them, in radian is
- π
 - $\pi/2$
 - $\pi/4$
 - zero
11. The KE of a particle executing SHM is maximum when its displacement is equal to
- amplitude
 - $1/4$ (amplitude)
 - $1/2$ amplitude
 - zero
12. If length of a pendulum is increased by 21%, its time period increases by
- 10%
 - 21%
 - 11%
 - 9.8%
13. A mass 'M' is suspended from a light spring. An additional mass 'm' when added displaces the spring further by a distance x . Now the combined mass will oscillate with a period of
- $2\pi \sqrt{\frac{mg}{x(M+m)}}$
 - $2\pi \sqrt{\frac{x(M+m)}{mg}}$
 - $2\pi \sqrt{\frac{mgx}{(M+m)}}$
 - $2\pi \sqrt{\frac{M+m}{mgx}}$
14. A rubber ball filled with water is used as the bob of a simple pendulum. If a small hole is made in the ball, its time period
- decreases
 - first decreases, then increases
 - first increases, then decreases
 - increases
15. A pendulum clock is taken to top of mountain, it will
- gain time
 - lose time
 - give correct time
 - stop giving time
16. A wrist watch of a man falls from top of tower. On reaching the ground, the watch will be
- giving more time
 - giving less time
 - giving correct time
 - functionless
17. The time period of a simple pendulum is T . The pendulum is now charged positively and is allowed to oscillate above a negatively charged plate. The time period of the pendulum will be

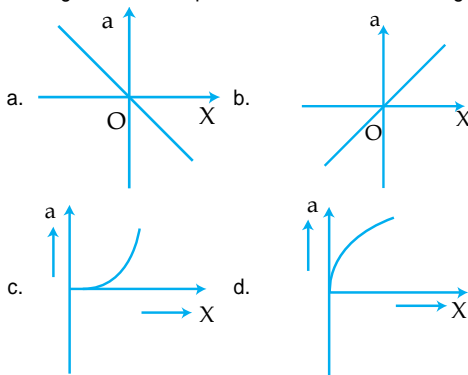
- T
- more than T
- less than T
- ∞

18. A mass M is suspended by a system of three springs A, B and C of force constants $2K$, K and K respectively as shown in figure. When mass M vibrates up and down, its time period is



- $2\pi \sqrt{\frac{M}{K}}$
- $2\pi \sqrt{\frac{M}{2K}}$
- $2\pi \sqrt{\frac{M}{4K}}$
- $2\pi \sqrt{\frac{2M}{K}}$

19. The variation of the acceleration (a) of the particle executing SHM with displacement x is as shown in figure



20. The displacement y (in cm) of a particle vibrating in SHM is given by, $y = 3 \sin \omega t + 4 \cos \omega t$. The amplitude of the vibrating particle is

- 3 cm
- 4 cm
- 5 cm
- 7 cm

Answers Key

1. c	2. b	3. c	4. b	5. b	6. d	7. d	8. b	9. c	10. b
11. d	12. a	13. b	14. c	15. a	16. c	17. c	18. a	19. a	20. c

B. Short Questions

- Can simple pendulum experiment be done inside a satellite?
- What do you mean by damped oscillation?
- When will the kinetic energy of a particle executing S.H.M be minimum ?
- When will the potential energy of a particle executing S.H.M. be maximum?


5. When will the kinetic energy of a particle executing S.H.M be maximum?
6. When will the potential energy of a particle executing S.H.M. be minimum?
7. What is forced oscillation?
8. What is free oscillation?
9. Define SHM?
10. A man with a wrist watch, on his hand falls from the top of a tower. Does the watch give correct time during the free fall?
11. What is the frequency/time period of a simple pendulum mounted in a cabin that is freely falling under gravity.
12. What are the characteristics of SHM?
13. Why does the motion of simple pendulum eventually stop?
14. How can one change time period of swing without changing its length?
15. The bob of a simple pendulum is made of ice. How will the time period change when the ice starts melting?
16. What is relation between uniform circular motion and S.H.M.?
17. When will the kinetic energy of a particle executing S.H.M be minimum and maximum?
18. When will the potential energy of a particle executing S.H.M. be maximum and minimum?
19. The length of a second pendulum on the surface of earth is 'l'. What will be the length on moon's surface?
20. What fraction of total energy is kinetic and what fraction is potential when the displacement is half of the amplitude?
21. Is there any effect of temperature in the time period of pendulum clock? How?

C. Long Questions

1. Define simple harmonic motion and obtain an expression of time period of a simple pendulum. [HSEB 2069]
2. What is simple pendulum? Show that motion of the bob of the simple pendulum is simple harmonic. Obtain an expression for its frequency. [HSEB 2065, B]
3. Define SHM? Derive a relation for total energy of a simple harmonic oscillator. [HSEB 2067 S]
4. Obtain an expression for the time period of a mass 'm' attached with a spring placed horizontally on a smooth table [HSEB 2063]
5. Define simple harmonic motion. Show that the oscillation of mass suspended from helical spring is simple harmonic. [HSEB 2061]
6. How does simple harmonic motion start? Derive the relation between the acceleration and displacement of the particle executing SHM. [HSEB 2055]
7. What are the characteristics of S.H.M? Derive an expression for the time period of such a motion.
8. Show that total energy of a particle executing S.H.M remains conserved.
9. Define amplitude, phase, time period and frequency of SHM.
10. What are characteristics of simple harmonic motion? Show that motion of vertical mass-spring system is simple harmonic and hence derive formula for its time period. [HSEB 2072 C]
11. Find an expression for total energy of particle in SHM and show that the particle obeys the law of conservation of energy. [HSEB 2070]

D. Numerical Questions

1. An object is undergoing simple harmonic motion with period $\frac{\pi}{2}$ second and amplitude 0.400 m. At $t = 0$, the object is at $x = 0$. How far is the object from the equilibrium position when $t = \pi/10$ second? [Ans: 0.380 m]

2. An object is vibrating with simple harmonic motion that has an amplitude of 18.0 cm and frequency of 6.00 Hz. Compute (a) the maximum magnitude of the acceleration and of the velocity; (b) the acceleration and speed when the object's coordinate is $x = +9.0$ cm. (c) the time required to move from the equilibrium position directly to a point 12.0 cm distant from it.
[Ans: (a) 256 ms^{-2} , 6.79 ms^{-1} , (b) -128 ms^{-2} , 5.588 ms^{-1} , (c) 0.0194 second]
3. If an object on a horizontal frictionless surface is attached to a spring, displaced and then released, it will oscillate. If it is displaced 0.120 m from its equilibrium position and released with zero initial speed, then after 0.800 sec, its displacement is found to be 0.120 m on the opposite side, and it has passed the equilibrium position once during this interval. Find (a) the amplitude, (b) the period and (c) the frequency.
[Ans: (a) 0.120 m, (b) 1.6 sec, (c) 0.625 Hz]
4. In a physics lab, you attach a 0.200 kg air track glider to the end of an ideal spring of negligible mass and it starts oscillating. The elapsed time from when the glider first moves through the equilibrium point to the second time it moves through that point is 2.60 second. Find the spring's force constant. [Ans: 0.292 N/m]
5. A simple pendulum 4 m long swings with an amplitude of 0.2 m.
a. Compute the velocity of the pendulum at its lowest point.
b. Compute its acceleration at the end of its path. [Ans: 0.32 m/s, 0.5 m/s^2]
6. A harmonic oscillator is made by using a 0.600 kg frictionless block and an ideal spring of unknown force constant. The oscillator is found to have a period of 0.150 sec. Find the force constant of the spring.
[Ans: 1052.8 Nm^{-1}]
7. A block with a mass of 3.00 kg hangs from an ideal spring of negligible mass. When displaced from equilibrium and released, the block oscillates with a period of 0.400 second. How much is the spring stretched when the block hangs in equilibrium (at rest)? [Ans: 3.97 cm]
8. On the earth, a certain simple pendulum has a period of 1.60 second. What is its period on the surface of the moon, where $g = 1.62 \text{ m/s}^2$. [Ans: 3.94 sec]
9. A particle of mass 0.25 kg vibrates with a period of 2.0 second. If its greatest displacement is 0.4m, what is its maximum kinetic energy? [Ans: 0.2 J]
10. A block with a mass of 3.00 kg is suspended from an ideal spring having negligible mass and stretches the spring 0.200 m. (a) What is the force constant of the spring? (b) What is the period of oscillation of the block if it is pulled down and released? [Ans: 147 N/m, 0.898 sec.]
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3 CHAPTER



Daniel Bernoulli
(1700-1782)

Fluid Statics

Syllabus:

Fluid statics: Pressure in a fluid; Buoyancy. Surface tension: Theory of surface tension; Surface energy. Angle of contact, capillarity and its applications. Fluid Dynamics: Newton's formula for viscosity in a liquid; Coefficient of viscosity. Poiseuille's formula and its application. Stokes law and its applications. Equation of continuity and its applications. Bernoulli's equation and its applications.



Learning Objectives

After studying this chapter, students should be able to:

- ✎ State and explain Archimedes principle and Pascal's law
- ✎ Define up-thrust, pressure in fluid, buoyancy, center of buoyancy and meta center
- ✎ State and use the law of floatation,
- ✎ Describe surface tension and explain its principle
- ✎ Establish the relation between surface energy and surface tension
- ✎ Define angle of contact and capillarity with examples
- ✎ State the Newton's Formula for viscosity of a liquid and define coefficient of viscosity
- ✎ Differentiate between laminar and turbulent flow & describe Reynolds number
- ✎ Recall and **use the Poiseuille's formula**
- ✎ State Stoke's law and use it to determine the coefficient of viscosity of given liquid
- ✎ Explain equation of continuity and its application
- ✎ Recall the Bernoulli's equation and explain its uses
- ✎ Solve the numerical problems and conceptual questions regarding the fluid statics

3.1 Introduction

Matter exists in three states like solids, liquids and gases. Fluids are those substances which have the property of flowing. Among the three states of matter, only liquids and gases are called fluids. The basic difference between solids and fluids are that, solids have their definite size and shape while a fluid has no definite shape of its own. Fluid can not oppose shearing stress for a long period. So that fluid has no modulus of rigidity but a fluid is characterized by bulk-modulus. Though, liquids and gases both are included in fluids, they can be distinguished from each other. A liquid is incompressible and possesses its own free surface while a gas is compressible and occupies whole volume available to it.

The **fluid statics** is the study of fluid at rest. So, its deals with the mechanical properties of the fluid in equilibrium situation. The characteristics properties of fluid may be described by its density and pressure.

3.2 Density, Relative Density or Specific Gravity

Density of a substance is defined as *"the ratio of its mass and its volume"*. If M and V be the mass and volume of substance (fluid) then, density (ρ) = $\frac{M}{V}$. Unit of density $\rho = \text{kg/m}^3$ or gm/cm^3 . Sometimes it is convenient to use relative density.

Relative density or specific gravity of a substance is defined as the ratio of the mass of certain volume of it to the mass of same volume of water at 4°C . So,

$$\begin{aligned}\text{Specific gravity of a body} &= \frac{\text{Mass of certain volume of a substance}}{\text{Mass of equal volume of water at } 4^\circ\text{C}} \\ &= \frac{\text{mass of 1 cc of a substance}}{\text{mass of 1cc of water at } 4^\circ\text{C}} = \frac{\rho}{\rho_w \text{ at } 4^\circ\text{C}} = \frac{\rho}{1\text{gm/cc}}\end{aligned}$$

Since, $\rho_w = 1\text{gm/cc}$ at 4°C , specific gravity of a substance is numerically equal to its density in cgs system. Again,

$$\begin{aligned}\text{Specific gravity of a body} &= \frac{\text{mass of certain volume of a substance}}{\text{mass of equal volume of water at } 4^\circ\text{C}} \\ &= \frac{\text{weight of certain volume of a substance}}{\text{weight of equal volume of water at } 4^\circ\text{C}} \\ &= \frac{\text{wt. of V.cc of a substance}}{\text{wt. of V.cc of water at room temp}} \times \frac{\text{wt.of V.cc of water at room temp.}}{\text{wt. of V.cc of water at } 4^\circ\text{C temp}} \\ &= \frac{\text{wt. of V.cc of a substance}}{\text{loss in wt.of substance in water}} \times \text{sp. of gravity of water at room temp}\end{aligned}$$

Measurement of Specific Gravity of Substances

Specific gravity of substances are measured by following ways.

1. **An object heavier than water ($\rho > \rho_w$):** Let, w_1 and w_2 are the weights of object in air and water respectively. Then, Specific gravity of object = $\frac{w_1}{w_1 - w_2} \times \text{Sp. gravity of water at room temperature}$
2. **An object that floats in water ($\rho < \rho_w$):** Consider, a cork floats in water is weighted in air is w_1 . Take another object which can sink in water is tied on the first object and with sinker in water and the

object in air, weight of both system is taken. Let, w_2 be the weight of the object in air and sinker in water and w_3 is the weight of object and sinker both in water. Therefore,
loss of weight of object in water = $w_2 - w_3$.

$$\text{Sp. gravity of object} = \frac{w_1}{w_2 - w_3} \times \text{Sp. gravity of water at room temperature}$$

3. Sp. gravity of a liquid: A body which can sink in the liquid and in water is taken.

Let, its weight in air be w_1 .

$$\text{Weight of the solid in water} = w_2$$

$$\text{Weight of the solid in liquid} = w_3$$

$$\text{Weight of Vcc. of liquid} = w_1 - w_2$$

$$\text{Weight of Vcc. of water} = w_1 - w_3$$

$$\text{Sp. gravity of liquid} = \frac{\text{wt. of certain volume of liquid}}{\text{wt. of equal volume of water}} \times \text{Sp. gravity of water at room temperature}$$

$$\text{Therefore, specific gravity} = \frac{w_1 - w_3}{w_1 - w_2} \times \text{Sp. gravity of water at room temperature}$$

4. Specific gravity of object soluble in water:

Let, weight of object soluble (salt) in air = w_1

$$\text{Wt. of the salt in Kerosene} = w_2$$

$$\text{Wt. of displaced Kerosene} = w_1 - w_2$$

Take a solid sinker which is insoluble in water and in Kerosene.

Let, Wt. of solid in air = w_3

$$\text{Wt. of solid in water} = w_4$$

$$\text{Wt. of solid in Kerosene} = w_5$$

$$\text{Wt. of certain volume of water} = w_3 - w_4$$

$$\text{Wt. of equal volume of water} = w_3 - w_5$$

$$\text{Therefore, specific gravity of salt} = \frac{w_1}{w_1 - w_2} \times \frac{w_3 - w_5}{w_3 - w_4} \times \text{specific gravity of water at room temperature}$$

3.3 Pressure and Different Types of Pressure

"The normal force acting per unit area on a surface is called the pressure." If F is the normal force acting on surface of area A , then pressure

$$P = \frac{F}{A} \quad \dots \quad (1)$$

The pressure depends on two factors. They are

1. Normal force applied and
2. Surface area

Unit: In S.I system, the unit of force is Newton and unit of area is m^2 . So,

$$\text{Pressure} = \frac{\text{Newton}}{\text{meter}^2} \text{ i.e., } \text{N/m}^2 \text{ or Pascal (pa) i.e., } 1 \text{ pa} = 1 \text{ Nm}^{-2}.$$

$$\text{Dimension of pressure: We have } P = \frac{F}{A} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$$

Different types of pressure are described below;

Pressure Due to a Fluid Column (Pressure in Fluid)

Fluid contained in a vessel exerts a pressure at its bottom and walls. Consider, a liquid contained in a cylindrical vessel (say). The height of liquid column 'h' and density of liquid is ρ . The area of vessel is A and acceleration due to gravity is g as shown in figure 3.1. The normal force acting on bottom of the vessel is equal to the weight of liquid column of height h. Therefore,

$$\text{Volume of liquid column} = \text{Area} \times \text{height} = Ah$$

$$\text{Mass of cylindrical column} = \text{volume} \times \text{density} = Ah\rho$$

$$\text{Weight of liquid column} = Mg = Ah\rho g$$

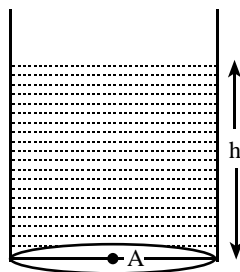
$$\text{Pressure of liquid column at bottom} = \frac{\text{Force or weight}}{\text{area}}$$

$$\text{or, } P = \frac{Ah\rho g}{A} = \rho gh$$

Thus, pressure of liquid column is, $P = \rho gh$

Pressure of liquid column depends on the following factors.

- i. Height of liquid column (h)
- ii. Acceleration due to gravity (g)
- iii. Density of liquid (ρ)



[Fig. 3.1, Water in beaker exerts pressure at the bottom]

Atmospheric Pressure and Standard Atmospheric Pressure

The weight exerted by vertical column of all layers of air of atmosphere on unit area of earth's surface is called the atmosphere pressure.

The atmospheric pressure changes with change in temperature, height and latitude. The standard atmospheric pressure is taken as 76 cm or 0.76 m of mercury column at 45°, latitude at sea level and at a temperature of 0°C. The density of mercury at 0°C is $13.6 \times 10^3 \text{ kg/m}^3$ and the value of g at 45° latitude at sea level is 9.8 m/sec^2 . Therefore, standard atmospheric pressure = pressure of 0.76 m of mercury column = $\rho gh = 13.6 \times 10^3 \times 9.8 \times 0.76 = 1.013 \times 10^5 \text{ N/m}^2$.

Note: It may be pointed out that the atmospheric pressure at the sea level is not always 760mm of Hg.

(i) Usually pressure is measured in terms of height of mercury column i.e. 1cm of Hg = 10^{-2} meter of Hg = $1.33 \times 10^3 \text{ Pa}$.

(ii) 76cm height of mercury column is called 1 atmosphere So, 1 atm = 76cm of Hg

i.e., 1 atm = $76 \times 1.33 \times 10^3 \text{ Pa} = 1.013 \times 10^5 \text{ Pa}$.

3.4 Upthrust or Buoyancy

"The upward force exerted by a fluid on an object which is completely or partially immersed in the fluid is called the upthrust or buoyancy."

When a body is immersed in a fluid, the fluid exerts pressure on all faces of the body. The fluid pressure increases with depth, $P = \rho gh$. The upward thrust at the bottom is more than downward thrust on the top (since the bottom is at the greater depth than the top). Hence, a resultant upward force acts on the body. The upward force acting on a body immersed in a fluid is called upthrust or buoyancy force and the phenomenon is called buoyancy. The force of buoyancy acts through the centre of gravity of the displaced fluid which is called centre of buoyancy.

3.5 Archimedes Principle

Archimedes principle states that "When a body is fully or partially immersed in a fluid, it experiences an upthrust which is equal to the weight of the fluid displaced by the body."

Suppose, a body has weight $W_1 = m_1g$ in air and volume V . Let, the weight of the body in liquid be $W_2 = m_2g$. Then loss in weight of the body in liquid or upthrust

$$W = W_1 - W_2 = m_1g - m_2g$$

$$\text{Therefore, upthrust (u)} = mg \quad \dots (1) \quad \text{since, } m_1 - m_2 = m$$

Let, A , h and ρ be the cross-section area, thickness when a body completely immersed in a fluid and density of a body respectively. Suppose, h_1 and h_2 be the depth of upper face and lower face of the body from the surface of the fluid as shown in figure 3.2.

Let, P_{atm} be the atmospheric pressure. Total downward thrust on the top face of a body

$$F_1 = (P_{\text{atm}} + h_1\rho g)A \quad \dots (2)$$

Total downward thrust on the bottom face of a body

$$F_2 = (P_{\text{atm}} + h_2\rho g)A \quad \dots (3)$$

$$\text{Resultant upthrust } F = F_2 - F_1 \quad \dots (4)$$

Using equation (2) and (3) in equation (4) we get,

$$F = h_2\rho gA - h_1\rho gA = A(h_2 - h_1)\rho g = Ah\rho g$$

$$\therefore F = V\rho g \quad \dots (5) \quad \text{where } V = Ah$$

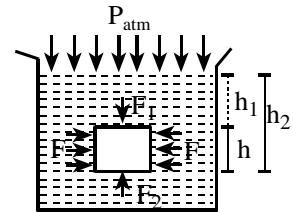
Here, V is the volume of a body which is equal to the volume of fluid displaced by the body.

Hence, $m = V\rho$ gives the mass of fluid displaced. Thus,

$$\text{Resultant upthrust} = mg = \text{weight of fluid displaced.} \dots (6)$$

Therefore, from equation (1) and (6) we get,

$$\text{Upthrust} = \text{weight of fluid displaced} \quad \dots (7)$$

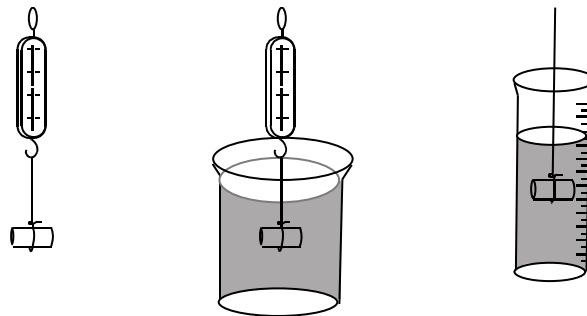


[Fig: 3.2, Archimedes principle]

Experimental Verification

We can verify Archimedes principle experimentally by following steps.

1. At first a solid body is weighted in air. Its weight be W_1 .
2. Again measure weight of the body in liquid by immersing it in the liquid in a beaker without touching the sides and bottom of the beaker. Let, its weight be W_2 in the liquid as shown in figure 3.3.



[Fig. 3.3, Verification of Archimedes Principle]

3. The difference in weight or loss of weight is, $W = W_1 - W_2 \dots (1)$
 4. Note the initial level of a liquid in a measuring cylinder.
 5. Immersed a body in the liquid and then note the final liquid level in it.
 6. The difference in liquid levels gives the volume of displaced liquid V which is equal to the volume of the body.
 7. Therefore, the weight of displaced liquid $= V\rho g \dots (2)$. Where, ρ is the density of the fluid.
- From equations (1) and (2), we get

$$W_1 - W_2 = V\rho g$$

$$\text{or, } m_1g - m_2g = mg$$

$$\text{or, } (m_1 - m_2)g = mg$$

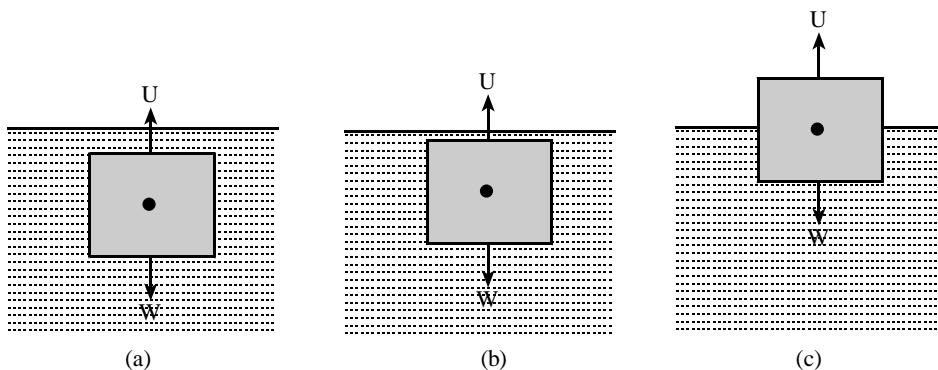
$$\therefore mg = mg. \text{ Since } m_1 - m_2 = m \text{ say.}$$

In this way, Archimedes principles is verified.

3.6 Principle of Flotation

Principle of flotation states that "*The weight of a floating body is equal to the weight of liquid displaced.*" Consider, a body is floating, its weight is W_1 , the volume of liquid displaced is V and density of liquid is ρ . Then weight of liquid displaced is, $W_2 = \rho vg$. Thus, for floatation of a body in a fluid or liquid, we must have, $W_1 = W_2$

Some cases



[Fig. 3.4, (a) A body is completely inside the liquid (b) body inside the liquid (c) A body floating in the liquid]

Case 1: If a body is immersed in a liquid completely, its weight W acts vertically downward and the upthrust U due to displaced liquid acts vertically upward. If the weight of body of volume V is greater than the upthrust, the body will sink in the liquid as shown in figure 3.4(a)

$$\text{i.e., } W > U$$

$$\text{or, } \rho Vg > \rho_l Vg$$

$$\text{or, } \rho > \rho_l. \text{ Where, } \rho = \text{be the density of sink body and } \rho_l \text{ be the density of liquid.}$$

Case 2: If the weight of a body of volume V is equal to the upthrust, the body just sinks and remains inside the liquid with its upper surface near the liquid surface as shown in figure 3.4(b).

$$\text{i.e., } W = U$$

$$\text{or, } \rho Vg = \rho_l Vg$$

or, $\rho = \rho_1$. Hence, density of liquid displaced is equal to density of a body in the liquid.

Case 3: If the weight of the body is smaller than the upthrust, the body will float in the liquid as shown in figure 3.4(c)

i.e., $W < U$

or, $\rho V g < \rho_l V g$

or, $\rho < \rho_l$. Hence, density of liquid displaced is greater than density of the body in the liquid.

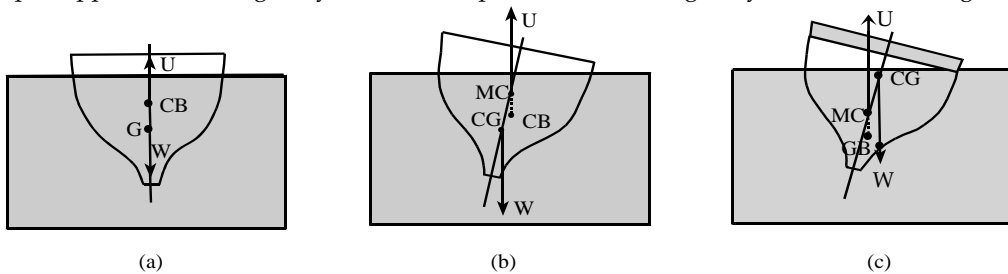
3.7 Equilibrium of Floating Bodies

The centre of gravity of displaced liquid is called centre of buoyancy (C.B). There are two conditions for the equilibrium of a floating body. They are

1. Weight of the liquid displaced must be equal to the weight of the body.
2. The centre of gravity of the body (C.G) and centre of buoyancy (C.B) must lie on the same vertical line.

If the floating body is slightly tilted from its equilibrium position, then the centre of gravity (C.G) and C.B. will not lie on the same vertical line, as C.B. shifts away. The point of intersection of the vertical line passing through C.B. and original vertical line is called the meta centre (M.C) of the floating body on the liquid. The condition that the body regains its equilibrium position or falls in the liquid depends upon the position of M.C and C.G of the body. These possible cases are discussed below:

- a. If the metacentre (M.C) lies above the centre of gravity C.G, the couple of forces (U and W) tends to rotate the body back to its original position. In this condition, the floating body is in stable equilibrium.
- b. If the metacentre (M.C.) lies below the C.G, the couple of forces (U and W) tends to rotate the body away from the original position. In this condition, floating body is in unstable equilibrium. The couple topples the floating body. The various positions of floating body are as shown in figure 3.5.



[Fig. 3.5, (a) Equilibrium position of a floating body (b) A body with heavy bottom regions its equilibrium position when it is tilted (c) A body with heavy top loses its equilibrium when it is tilted.]

Some Examples of Floatation

- **Balloons:** A hydrogen filled balloon is lighter than air and the air forces up it to a height where weight of balloon is equal to the upthrust of air there.
- **Iceberg:** Density of ice is smaller than the density of water. So, it floats in sea with certain volume out of water level.
- **Ships:** Though, the density of the materials used in ship is greater than the density of water, the structure of the ship is made such that it displaces more volume of water than it does in solid form. The construction of ship makes it floating as it is hollow inside it.

3.8 Types of Molecular Forces

According to the molecular theory, intermolecular forces are of two types. They are given below.

Cohesive force: The force of attraction between the molecules of the same substance is known as cohesive force or force of cohesion. It is maximum in solids, lesser in liquids and least in gases. Due to the strong cohesive force, solids have definite shape and resist all deforming forces, but due to less cohesive force, liquids and gases are flowing. Among the liquid, mercury has comparatively high cohesive force. Therefore, mercury does not wet the glass as force of cohesion dominates the effects of force of adhesion between mercury and glass.

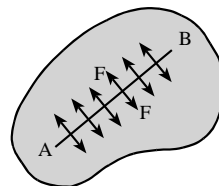
Adhesive force: The force of attraction between the molecules of two different substances is known as adhesive force or force of adhesion. Due to the high adhesive force, ink sticks to paper while writing. Water wets the wall of glass container because the force of adhesion between water and glass is greater than the force of cohesion between water molecules. Strong adhesion is shown by the materials like favicol, cement, glue etc.

3.9 Surface Tension

"The property of the liquid in which the surface acts like a stretched member and tends to occupy the minimum surface area is called surface tension." This property of liquid is responsible to hold the dense material on its surface. For examples, some insects can walk on the surface of water their feet making depression in the surface but not penetrating it. A sewing needle, if placed carefully on a water surface makes a small depression in the surface and floats on its surface even though its density much greater than that of water. There are so many other examples in which body whose density greater than the liquid (water) are observed floating on its surface. Thus, above examples concluded that surface of liquid seems like an elastic stretched membrane.

"The surface tension of a liquid is measured by the force per unit length on either side of an imaginary line drawn on liquid surface and the direction of force is perpendicular to line in the plane of surface."

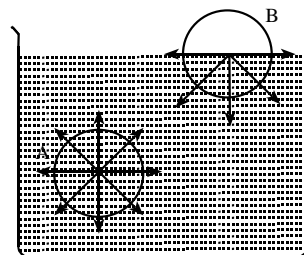
Suppose, AB is an imaginary line on liquid surface. The surface on one side exerts a pulling force on the other side which is perpendicular to line AB and in the plane of surface. Let length of AB is L and force on it is F as shown in figure 3.6. Then, surface tension $T = \frac{F}{L}$. The unit and dimension of surface tension are Newton/meter (N/m) and $[MT^{-2}]$ respectively.



[Fig. 3.6, Surface tension on the free surface of a liquid]

3.10 Molecular Theory of Surface Tension

The molecules of a liquid attract each other with a force of cohesion. Consider, a molecule A of a liquid lying well below the free surface of a liquid. It has sphere of influence of radius of the order 10^{-9} m. As this molecule is attracted by the neighbouring molecules lying within the sphere of influence as shown in figure 3.7, the resultant force due to all the molecules on A is zero. (Since



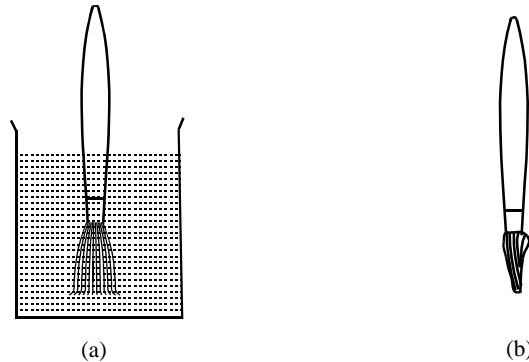
[Fig. 3.7, Molecular forces in liquid]

neighbouring molecules exert equal forces on this molecule A in all directions hence, resultant force acting on it is zero). Consider, molecule B on the surface of the liquid. Since, there are few molecules of liquid in vapour state above the free surface, the molecules B experiences forces of attraction only due to the molecules lying in the lower half of the sphere of influence. The resultant of all these forces is downward force. So, the molecules on the surface experiences maximum downward force.

When any molecule is brought towards the surface from the interior of liquid surface. Some work has to be done against the cohesive force. This work done on the molecules is stored in it in the form of potential energy. For equilibrium, a system must have minimum potential energy. So, there must be minimum number of molecules on the liquid surface. That is why, the liquid surface contracts like a stretched elastic membrane.

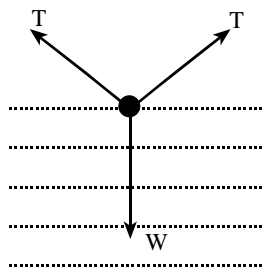
Some Examples Explaining Surface Tension

1. **Some insects float on water:** Insects bend their legs on the surface of water such that the deformed surface gives rise to the forces of surface tension which acts tangential to the deformed surfaces. The weight of the insect is balanced by the upward components of these forces of surface tension.
2. **The hair of a paint brush spreads into the water but cling together when it is taken out:** When the brush is in water, there is water all around its hair, consequently there is no free surface and the effect of surface tension is absent as shown in figure 3.8 (a). However, when the brush is taken out from water, the forces of surface tension contract the hair as shown in figure 3.8 (b).



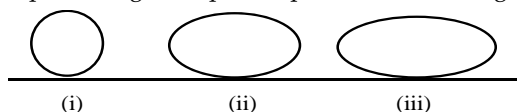
[Fig.3.8, (a) The hair of the brush in water spread out (b) The hair of brush cling together]

3. **Floating needle on water surface:** The water surface below the needle is gently depressed. The force of surface tension does not act horizontally but along in inclined direction. The vertical component of the surface tension balances the weight of the needle as shown in figure 3.9.



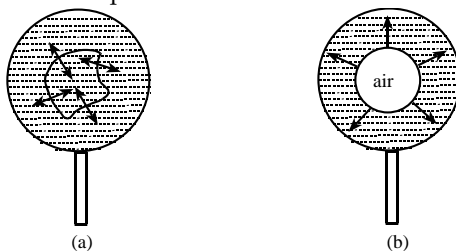
[Fig: 3.9, Needle floating on water]

4. **Small rain drops are spherical in shape:** The surface tension always tends to minimize the surface area of the liquid. The surface area is minimum on sphere for a given volume. But the effect of gravity makes the oval shape for larger drops of liquid as shown in figure 3.10.



[Fig: 3.10, (i) smallest drop (ii) intermediate drop (iii) largest drop.]

5. **Oil has less surface tension than water:** When a drop of oil is dropped on the surface of water, due to higher surface tension of water, the oil is stretched in all directions as a thin film. Mosquitoes breed on the free surface of stagnant water. Due to the surface tension, the liquid layer supports the eggs laid by the mosquitoes. When oil is spread in water, there is little surface tension and mosquitoes cannot breed.
6. **Circular endless wet thread on a soap film:** If we take a circular frame of a stiff wire and dip into a soap solution, a thin soap film is formed on the frame. If a wet endless thread loop is gently placed over the film, it takes any irregular shape as shown in figure 3.11 (a). But when the film is pricked at the centre, the loop is stretched outward and take a symmetrical circular shape is shown in figure 3.11 (b). This is because, for a given length a circle has the maximum surface area and so the outer liquid film tries to occupy maximum possible area like stretched elastic membrane.



[Fig. 3.11, (a) A loop of thread on a liquid surface, (b) A hole made at the centre of the circular loop]

Factor Affecting the Surface Tension

Surface tension is affected by the following factors.

1. **Mixing impurities:** When the impurities are mixed to the liquid, its surface tension decreases. For example, detergents and soap solutions are used to wash clothes. They decrease the surface tension of water and result in greater wetting and hence washing power is increased.
2. **Temperature:** The surface tension and temperature are inversely proportional to each other i.e., surface tension of liquid decreases on increasing temperature. Therefore, oil spreads on cold water but remains as a drop on hot water.
3. **Mixing highly soluble contamination:** A highly soluble substance increases the surface tension. For examples, sodium chloride.

Note:

1. Movement of insects is possible due to the force of reaction produced by surface tension.
2. Hot soup spreads over the tongue easily, hence it is tastier than cold soup because the surface tension decreases with rise in temperature.
3. Soap washes clothes effectively because the surface tension of soap solution is low, so it can spread over a large area.
4. Lubricating oils and paints spread properly because they have low surface tension.
5. Surface tension of oil is smaller than that of water so oil spreads on water. This property is used to calm down the stormy waves in sea.
6. Antiseptics spread over a large area and help to cure the wounds because they have low surface tension.

3.11 Surface Energy

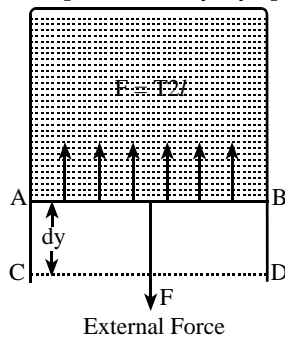
When the surface area of a liquid is increased, some molecules are brought from the interior to the surface. This requires the work against the cohesive forces among molecules in the surface film. This work is stored in the form of potential energy of these molecules. Thus, the molecules lying in the surface possess potential energy than other molecules. This excess energy per unit area of the free surface of the liquid is called the surface energy.

Relation between Surface Energy and Surface Tension

Consider a wire is bent in U - shape and another piece of wire AB is used to slide over it. Now this device is dipped in soap solution and pulled out. A soap film is formed between wire frame. If l is the length of wire AB and T be the surface tension of soap solution, then force of surface tension on wire AB in upward direction is given by

$$F = T.2l \quad \dots (1)$$

Since, the soap film touches the wires above as well as below, the length is taken twice. If now movable wire AB is pulled down slowly through a displacement ' dy ' by applying a force F as shown in figure 3.12.



[Fig. 3.12, Surface energy]

The work done against surface tension

$$W = F.dy \quad [\because 2l.dy = dA]$$

$$\text{or, } W = T.2l.dy$$

$$\text{or, } W = T(2l.dy) \quad \dots (2)$$

Here $2l.dy$ is increased in area of both vertical films between wire AB and liquid = dA

Therefore, equation (2) becomes,

$$W = T.dA$$

$$T = \frac{W}{dA} \quad \dots (3).$$

If $A = 1\text{m}^2$ and $T = W$

Thus, surface tension of a liquid is numerically equal to the work required to increase unit surface area of liquid film at constant temperature. i.e., surface tension T is numerically equal to surface energy W .

[Note: Surface tension arises due to intermolecular forces, it is independent of 'gravity'.]

3.12 Angle of Contact (θ)

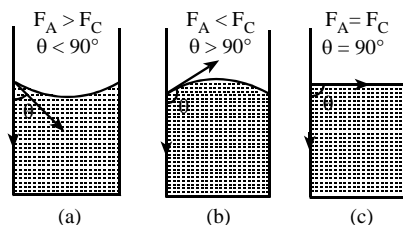
When the free surface of a liquid comes in contact with a solid, then the surface of liquid becomes curved at the point of contact. Thus, "*The angle between tangent drawn at the point of contact to solid surface within the liquid and liquid surface is called the angle of contact.*" It is denoted by θ . Its value depends upon the relative magnitudes of adhesive force between solid - liquid molecules and cohesive force between liquid molecular.

Case I: If adhesive force between solid-liquid molecules is greater than cohesive force between liquid molecules (i.e., $F_A > F_C$), the liquid wets the solid and the angle of contact is acute ($\theta < 90^\circ$) as shown in figure 3.13 (a). Angle of contact is 0° for pure water and clean glass but nearly 8° for ordinary water and glass.

Case II : If adhesive force between solid-liquid molecules is less than cohesive force between liquid molecules (i.e., $F_A < F_C$), the liquid does not wet the solid, angle of contact is obtuse ($\theta > 90^\circ$) as shown in figure 3.13 (b). For mercury and glass, angle of contact is 135° .

Case III : If adhesive force between solid - liquid molecules is equal to the cohesive force between liquid molecules, (i.e., $F_A = F_C$) the angle of contact is 90° as shown in figure 3.13 (c). For example; for pure water and silver angle of contact is 90° .

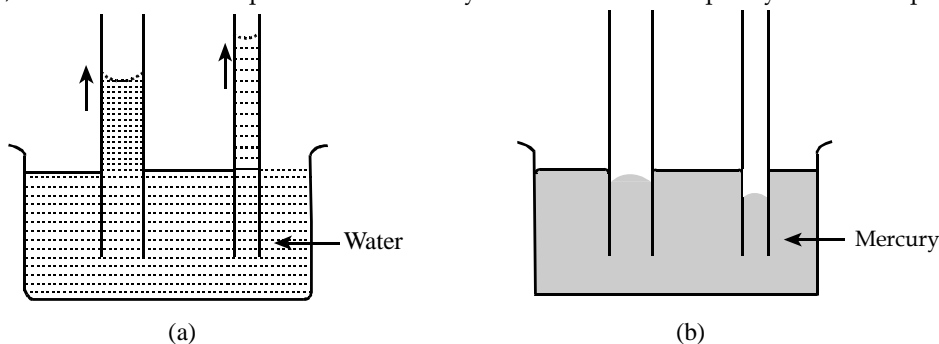
Note : The surface of liquid is usually curved where it is in contact with a solid. The curved surface of the liquid is called meniscus. The shape of the meniscus is determined by the relative strengths of cohesive and adhesive forces acting on the molecules. The concave meniscus is called lower meniscus and convex meniscus is called upper meniscus.



[Fig. 3.13, Showing angle of contact]

3.13 Capillary Tube and Capillary Action (Capillarity)

"A tube of very fine bore whose diameter is comparable with the hair is called capillary tube." When a glass capillary tube opened at both ends is dipped vertically in water, the water in the vessel as shown in figure 3.14 (a). But, in the case of mercury, the liquid is depressed in the tube below the level of mercury in the vessel as shown in figure 3.14(b). These rise or fall of liquid depends on the angle of contact. If a liquid whose angle of contact is less than 90° , suffers capillary rise and liquid whose angle of contact is greater than 90° suffers capillary depression. If $\theta = 90^\circ$, the liquid will neither rise nor fall. Therefore, this rise or fall of a liquid in a tube of very fine bore is called capillary action or capillarity.



[Fig. 3.14, Capillary rise and fall]

Some Examples of Capillary Action

1. We use towels to dry our body after taking bath.
2. Oil rises in cotton wicks of lamps through the small capillaries between the threads.
3. Sap in plants rises by capillary action.
4. The tip of the nib of a pen is split to provide capillary action for rise of ink.
5. A blotting paper absorbs ink by capillary action.
6. Sand is drier soil than clay because holes between the sand particles are not so fine as to draw up water by capillary action etc.

3.14 Viscosity

The branch of mechanics which deals with study of motion of objects by considering their causes of motion is called dynamics. Fluid dynamics describes the motion of any object in the fluid (liquid or gas) medium. The term related to describe the fluid dynamics is viscosity.

Viscosity is internal friction in a fluid. Viscous forces oppose the motion of one portion of a fluid relative to another. Thus, the property of viscosity of liquid is related to this frictional force. So that the frictional force exerted by liquid is called the viscous force. Some examples;

- When we swim in a swimming pool, we feel a force exerted by water opposite to the direction of motion.
- When we stir a liquid and leave, the liquid comes to rest after some time.

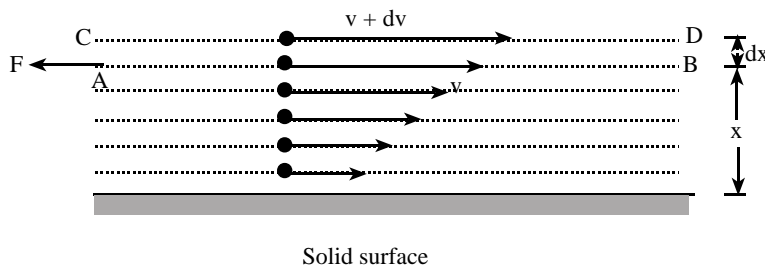
These examples clearly show that some type of frictional force acts between liquid layers.

Therefore, *"the property of a fluid (liquid or gas) by virtue of which an internal friction comes into play when the fluid is in motion and opposes the relative motion of its different layers is called viscosity."*

3.15 Newton's Formula for Viscosity and Coefficient of Viscosity

When a liquid flows on a horizontal fixed surface, the velocity of liquid - layer in contact with fixed solid surface is minimum (zero) and increases as a distance of liquid layers from fixed surface increases.

Consider, two layers AB and CD moving with velocities v and $(v + dv)$ at a distance x and $(x + dx)$ respectively from the fixed solid surface as shown in figure 3.15.



[Fig. 3.15, Different layers of liquid flowing over the solid surface]

The rate of change of velocity of liquid layers with distance in a direction perpendicular to direction of flow of liquid is called velocity gradient,

$$\text{i.e., velocity gradient} = \frac{dv}{dx}$$

According to Newton, the viscous force F depends upon the following factors.

- i. It is directly proportional to area A of the layers in contact, i.e.,

$$F \propto A \quad \dots (1)$$
- ii. It is directly proportional to the velocity gradient between the layers i.e.,

$$F \propto \frac{dv}{dx} \quad \dots (2)$$

Combining equation (1) and (2) we get,

$$F \propto A \frac{dv}{dx}$$

$$\text{or, } F = -\eta A \frac{dv}{dx} \quad \dots (3).$$

Where η is constant of proportionality and is called coefficient of viscosity. The negative sign shows that the direction of viscous force F is opposite to the direction of motion of the fluid. The value of coefficient of viscosity depends upon the nature of the liquid.

If $A = 1$ and $\frac{dv}{dx} = 1$, then equation (3) will be

$$F = -\eta.$$

Hence, coefficient of viscosity of a liquid is defined as "the viscous drag or viscous force acting per unit area of the layer having unit velocity gradient perpendicular to the direction of the flow of the liquid."

Units of η :

We have,

$$F = -\eta A \frac{dv}{dx} \Rightarrow \eta = -\frac{F dx}{A dv}$$

$$\text{In S.I. system, } \eta = \frac{\text{N m}}{\text{m}^2 \text{m/sec}} = \text{N sec m}^{-2} = \text{kgm}^{-1} \text{sec}^{-1}$$

In CGS system, $\eta = \text{gcm}^{-1}\text{sec}^{-1}$ is called poise.

$$\text{i.e., } 1 \text{ poise} = \frac{1 \text{ dyne}}{1 \text{ cm}^2 \times \left(\frac{1 \text{ cms}^{-1}}{\text{cm}}\right)} = 1 \text{ dyne cm}^{-2}\text{sec}$$

Dimensions of η : We have, $\eta = \frac{F}{A \frac{dv}{dx}} = \frac{[MLT^{-2}]}{[L^2] \left[\frac{LT^{-1}}{L} \right]} = [ML^{-1}T^{-1}]$

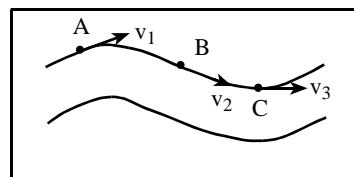
3.16 Streamline or Laminar Flow and Turbulent Flow

"When the flow of liquid is such that the velocity v of every particle at any point of the fluid is constant, then the flow is said to be steady or streamline or laminar flow.

OR

The regular flow of a liquid in which each particle follows the path and motion of its preceding particle is called the streamline flow."

In laminar flow, a liquid is flowing over a horizontal surface in the form of layers of different steady velocities which do not mix each other. Consider a liquid flowing through a tube as shown in figure 3.16. Let, the path followed by the particles of the liquid is represented by a line ABC.

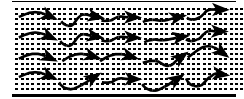


[Fig. 3.16, Stream-line flow]

Let, V_1 , V_2 and V_3 are the velocities of a particle of the liquid at point A, B and C respectively on its path. If all the succeeding particles of the fluid move along ABC with velocities V_1, V_2 and V_3 at A, B and C, then flow of liquid is known as steady or streamlines flow. The streamline may be in the form of a straight line or a curve. The tangent drawn at any point on the stream line gives the direction of velocity of liquid at that point. Clearly, the two stream lines cannot intersect each other because if they do so, there will be two directions of the velocity of liquid at that point which is impossible.

When the fluid moves with a velocity greater than a certain velocity (critical velocity), the motion of the particles of fluid (liquid or gas) becomes disorderly or irregular, then such a flow is called Turbulent flow.

The turbulent flow of a fluid (liquid) is as shown in figure 3.17. The path and velocity of the particles of fluid changes continuously and haphazardly with time from point to point in turbulent flow. For examples, the air flow behind a moving bus or train, the flow of water just behind boat or ship, the smoke from a cigarette after rising a short distance, etc.



$$v < v_c$$

[Fig. 3.17, Turbulent flow]

Critical Velocity

"The motion of a liquid in a tube remains streamline if its velocity is less than certain velocity, called its critical velocity". It is denoted by V_c . Its value for a liquid depends on density of liquid (ρ), viscosity (η) of liquid and diameter (D) of capillary tube.

We can show that, $V_c = \frac{k\eta}{\rho D}$

where K is a constant called Reynold's constant and for narrow tube its value is 2000.

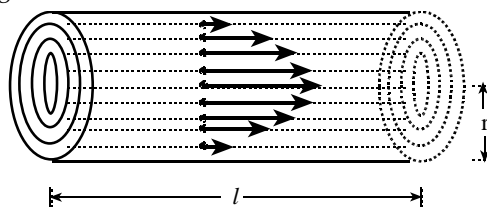
Reynold's Number

"Reynold's number is purely numeric and determines the maximum velocity of liquid flow for streamline (or laminar) flow". Its maximum value is 2000. Reynold's number is given by $K = \frac{\rho D V_c}{\eta}$. Its value is upto 2000 for streamline flow. If it exceeds above 3000, the flow is turbulent.

Note: Physically, the Reynold's number is the ratio of inertial force per unit area to the viscous force per unit area. For maximum value of V for streamline flow (i.e., $V = V_c$), this number is called the critical Reynold number. Thus critical Reynold number is, $K = \frac{V_c \rho D}{\eta}$

3.17 Poiseuille's Formula

The streamline flow of liquid in capillary tube studied by poiseuille. The streamline flow of liquid in a capillary tube as shown in figure 3.18.



[Fig. 3.18, Stream line flow of a liquid in a capillary tube]

Poiseuille's concluded that the volume V of the liquid flowing per second through a capillary tube is

1. Directly proportional to the difference of pressure P between the two ends of the tube,

$$\text{i.e., } V \propto P \quad \dots (1)$$

2. Directly proportional to the fourth power of radius r of the capillary tube.

$$\text{i.e., } V \propto r^4 \quad \dots (2)$$

3. Inversely proportional to the coefficient of viscosity η of the liquid.

$$\text{i.e., } V \propto \frac{1}{\eta} \quad \dots (3), \text{ and}$$

4. Inversely proportional to the length l of the capillary tube

$$\text{i.e., } V \propto \frac{1}{l} \quad \dots (4)$$

Combining equations (1), (2) (3) and (4) we get,

$$V \propto \frac{Pr^4}{\eta l}$$

$$\text{or, } V = \frac{Kpr^4}{\eta l}. \text{ Where } K = \frac{\pi}{8} \text{ is called proportionality constant.}$$

$$\therefore V = \frac{\pi pr^4}{8\eta l}. \text{ This is called Poiseuille's formula.}$$

3.18 Derivation of Poiseuille's Formula by Dimensional Method

Poiseuille's found that the volume of liquid flowing per second through a capillary tube depends on

- i. Pressure gradient $\left(\frac{P}{l}\right)$
- ii. Radius (r) of capillary tube, and
- iii. Coefficient of viscosity (η)

$$\text{Thus, } V \propto \left(\frac{P}{l}\right)^a r^b \eta^c$$

$$\text{or, } V = K \left(\frac{P}{l}\right)^a r^b \eta^c \quad \dots (1)$$

Where, K is a dimensionless constant. We know that

$$V = [L^3 T^{-1}]$$

$$\left(\frac{P}{l}\right) = \frac{[ML^{-1}T^{-2}]}{[L]} = [ML^{-2}T^{-2}]$$

$$(r) = [L]$$

$$\text{Dimension of coefficient of viscosity } \eta = [ML^{-1}T^{-1}]$$

Substituting dimensions of quantities in equation (1) we get,

$$[L^3 T^{-1}] = [ML^{-2}T^{-2}]^a [L]^b [ML^{-1}T^{-1}]^c$$

$$\text{or, } [M^0 L^3 T^{-1}] = [M^{a+c} L^{-2a+b-c} T^{-2a-c}]$$

Comparing dimensions of M , L and T on either side we get,

$$\left. \begin{array}{l} a + c = 0 \\ -2a + b - c = 3 \end{array} \right\} \quad \dots (2)$$

$$-2a - c = -1$$

Solving equation (2) we get,

$$a = 1, b = 4, c = -1$$

Therefore, equation (1) will be,

$$V = K \left(\frac{p}{l} \right)^1 r^4 \eta^{-1}$$

$$\text{or, } V = \frac{K p r^4}{\eta l}$$

By experiments, we can show that, $K = \frac{\pi}{8}$. Thus,

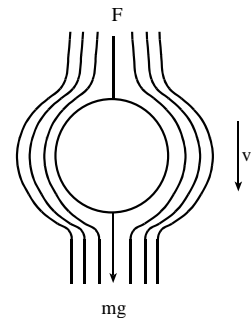
$$V = \frac{\pi p r^4}{8 \eta l} \quad \dots (3)$$

This is poiseuille's formula.

3.19 Terminal Velocity

"The constant maximum velocity acquired by a body while falling through a viscous fluid (medium) is called terminal velocity." It is denoted by v_t .

Consider, a small spherical body falls through a viscous fluid, the layer of fluid in contact with the body also moves with the same velocity. But, the layers of fluid at a large distance from the falling body remain undisturbed. Thus, the falling body produces a relative motion between different layers of the fluid. The relative velocity of the body increases the opposing force also increases. It is found that the body after attaining a certain velocity starts moving with a constant velocity in the fluid. This constant velocity of a body while moving in a fluid is called terminal velocity. The figure 3.19, shows the falling body produces a relative motion between different layers of the fluid.



[Fig. 3.19, Falling body produces a relative motion between different layers of the fluid]

3.20 Stoke's Law

The viscous force F acting on a spherical body of radius r moving with terminal velocity v in a fluid of coefficient of viscosity η is given by,

$$F = 6\pi\eta r v$$

which is called stoke's law.

Stoke's performed many experiments on small spherical bodies in different fluids. He concluded that the viscous force F acting on a small sphere depends on

1. Coefficient of viscosity η of the fluid.
2. Terminal velocity v_t of the spherical body
3. The radius r of sphere.

Therefore, $F \propto \eta v_t r$

$$\text{or, } F = K\eta^x v_t^y r^z \quad \dots (1).$$

Where, K is proportionality constant which is dimensionless. Comparing the corresponding dimensions of physical quantities we get,

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^x [LT^{-1}]^y [L]^z$$

$$\text{or, } [MLT^{-2}] = [M^x L^{-x+y+z} T^{-x-y}]$$

Equating corresponding powers of M, L and T on both sides,

$$x = 1,$$

$$\text{or, } -x + y + z = 1 \text{ and } -x - y = -2$$

Solving these equations, we get

$$x = 1, y = 1, z = 1$$

Using these values in equation (1) we get

$$F = K\eta v_t r \quad \dots (2)$$

Experimentally, the value of K is found as $K = 6\pi$

Thus, equation (2) can be written as,

$$F = 6\pi\eta r v_t \quad \dots (3)$$

This is Stoke's law.

Note: Stoke's law is valid under following assumptions. (i) The viscous medium is homogenous. (ii) The spherical body is perfectly rigid and smooth. (iii) The size of the moving body is much larger than the distance between the molecules of the medium. (iv) The body does not slip in the medium.

3.21 Experimental Determination of Coefficient of Viscosity of a Liquid

In an experiment, when a spherical ball falls freely through a viscous medium such as a liquid, its velocity at first goes on increasing. Therefore, the opposing viscous force which acts upward also goes on increasing. Finally a stage is reached *"at which the weight of the ball is just equal to the sum of the upthrust due to buoyancy and the upward viscous force. In such condition, the net force acting on the body is zero and the ball starts to fall with a constant velocity is called terminal velocity."*

Let, r be the radius of spherical ball falling through the viscous fluid of density σ and coefficient of viscosity η , ρ be the density of solid ball, the viscous forces acting on the body are

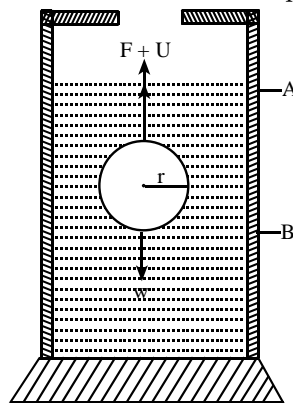
1. Its weight W in the downward direction.
2. Upward upthrust U equal to the weight of the displaced fluid.
3. The viscous force F in a direction opposite to the direction of motion of the body. So,

The downward force acting on the body = W

Upward force acting on the ball = $U + F$ [as shown in figure 3.20.]

When the spherical body attains the terminal velocity then,

$$W = U + F \quad \dots (1)$$



[Fig. 3.20, A metal sphere of radius r falls freely under gravity in a fluid]

We have, $F = 6\pi\eta r v_t$... (2)

The weight of the solid ball is,

$$W = \frac{4}{3} \pi r^3 \rho g \quad \dots (3)$$

Upthrust which is equal to the weight of the liquid displaced is given by

$$U = \frac{4}{3} \pi r^3 \sigma g \quad \dots (4)$$

Using equation (2), (3) and (4) in equation (1) we get,

$$\frac{4}{3} \pi r^3 \rho g = \frac{4}{3} \pi r^3 \sigma g + 6\pi\eta r v_t$$

$$\text{or, } \frac{4}{3} \pi r^3 (\rho - \sigma) g = 6\pi\eta r v_t$$

$$\text{or, } \eta = \frac{2r^2(\rho - \sigma)g}{9v_t} \quad \dots (5)$$

This equation (5) is an expression of coefficient of viscosity of the liquid or fluid. From equation (5), the value of η of a liquid can be determined.

3.22 Equation of Continuity

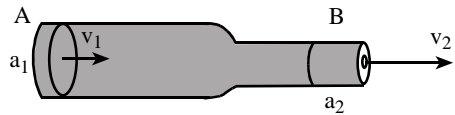
It states that *"If an ideal liquid (i.e., non viscous and incompressible) flows through a tube in streamline motion, then the product of cross-sectional area of tube and velocity of flow is same at every point in the tube."*

i.e., $av = \text{constant}$.

Where 'a' is cross-sectional area and 'v' is the velocity of flow of liquid at any point in the tube.

Proof: Consider, the steady flow of a non-viscous liquid through a pipe of varying cross-sectional area as shown in figure 3.21.

Let, a_1 , v_1 and ρ_1 are the area of cross section of the tube, velocity of flow of the liquid and density of the liquid respectively at point A of the tube. Similarly a_2 , v_2 and ρ_2 are the corresponding values at the point B of the tube.



[Fig. 3.21, Steady flow of liquid]

Volume of the liquid entering per second at A = $a_1 v_1$

Mass of liquid entering per second at A = $a_1 v_1 \rho_1$

Similarly, mass of liquid leaving per second at B = $a_2 v_2 \rho_2$

If there is no loss of liquid in the tube and the flow is steady, then,

Mass of liquid entering per second at A = mass of liquid leaving per second at B.

$$\therefore a_1 v_1 \rho_1 = a_2 v_2 \rho_2 \quad \dots (1)$$

If the liquid is incompressible then, $\rho_1 = \rho_2$. So that, equation (1) will be

$$a_1 v_1 = a_2 v_2 = \text{constant}$$

In general, $av = \text{constant}$. This is called equation of continuity.

3.23 Energy of a Liquid

A flowing liquid possesses three types of energies. They are

1. **Kinetic energy:** The energy stored due to motion of the flowing liquid is called its kinetic energy.

Consider, a liquid of mass m , volume V and density ρ flows with velocity v and then its K.E. is

$$\text{K.E.} = \frac{1}{2} mv^2 \quad \dots (1)$$

$$\text{K.E. per unit mass of liquid} = \frac{1}{2} v^2 \quad \dots (2)$$

2. **Potential energy:** The energy stored due to position of a flowing liquid is called its potential energy.

If a flowing liquid is at a height h , then potential energy of liquid of mass m is

$$\text{P.E.} = mgh \quad \dots (1)$$

$$\text{Potential energy of liquid per unit mass} = gh \quad \dots (2)$$

3. **Pressure energy:** The energy of flowing liquid due to pressure exerted on liquid is called pressure energy. Suppose, P is the pressure exerted on Cross-sectional area A of liquid and due to it the liquid flows through distance x .

Pressure energy = work done by liquid due to applied pressure.

$$= \text{Force} \times \text{displacement}$$

$$= \text{Pressure} \times \text{area} \times \text{displacement}$$

$$\text{Thus, pressure energy} = P Ax \quad \dots (1). \text{ But, } Ax = \text{Volume of liquid } V$$

$$\text{Then, pressure energy} = PV = P \frac{m}{\rho} \text{ where } V = \frac{m}{\rho}$$

$$\text{The pressure energy per unit mass of liquid is } = \frac{P}{\rho} \quad \dots (2)$$

Therefore, the total energy per unit mass of a liquid flowing with velocity v at a height h under pressure P is given by

$$E = \text{K.E.} + \text{P.E.} + E_P \quad [\text{Here, } E_P = \text{pressure energy}]$$

$$E = \frac{1}{2} v^2 + gh + \frac{P}{\rho}$$

This is an expression for total energy possessed by a liquid in motion.

3.24 Bernoulli's Theorem

The Swiss physicist Daniel Bernoulli in 1738, established an expression relating the pressure, flow speed and height for flow of an ideal, incompressible fluid, which is called Bernoulli's theorem. Therefore, Bernoulli's theorem states that "*for the streamline flow of an ideal fluid (non-viscous and incompressible), the sum of pressure energy, kinetic energy and potential energy per unit mass is always constant.*"

$$\text{i.e., } \frac{P}{\rho} + \frac{1}{2} v^2 + gh = \text{constant. This is called Bernoulli's equation.}$$

Derivation of Bernoulli's Equation

Consider, a tube PQ of varying area of cross-section through which an ideal liquid is in streamline flow as shown in figure 3.22. Let, P_1 , A_1 , h_1 , v_1 and P_2 , A_2 , h_2 , v_2 are the pressure, area of cross-section, height and velocity of flow at points P and Q respectively.

Force acting on the liquid at point P is $P_1 A_1$

Distance traveled by the liquid in time Δt is $v_1 \times \Delta t$.

Work done on the liquid due to the force $P_1 A_1$ = force \times displacement = $P_1 A_1 v_1 \Delta t$.

Work done by the fluid against pressure p_2 is $p_2 A_2 v_2 \Delta t$

The net work done on the liquid by the pressure energy in moving the liquid from the section P to Q is, $W = P_1 A_1 v_1 \Delta t - P_2 A_2 v_2 \Delta t$.

But from the equation of continuity, we have

$$A_1 v_1 \Delta t = A_2 v_2 \Delta t = V = \frac{m}{\rho}$$

So that, total work done on the liquid = $P_1 V - P_2 V = (P_1 - P_2) V$

$$= (P_1 - P_2) \frac{m}{\rho}$$

Where m be the mass of the fluid transferred. Thus, work done by the pressure energy on the fluid increases the K.E. and P.E. of the liquid when it flows from P to Q.

Increase in P.E. of the liquid = $mgh_2 - mgh_1$

Similarly, the increase in K.E. of the liquid = $\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

According to work energy theorem,

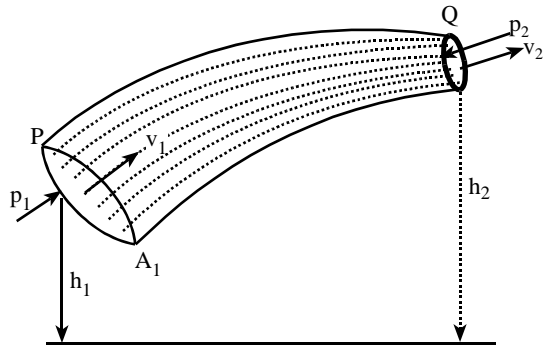
Work done by the pressure energy = increase in P.E + increases in K.E.

$$\therefore (P_1 - P_2) \frac{m}{\rho} = mg(h_2 - h_1) + \frac{1}{2} m (v_2^2 - v_1^2)$$

$$\text{or, } \frac{P_1}{\rho} - \frac{P_2}{\rho} = gh_2 - gh_1 + \frac{1}{2} v_2^2 - \frac{1}{2} v_1^2$$

$$\text{or, } \frac{P_1}{\rho} + gh_1 + \frac{1}{2} v_1^2 = \frac{P_2}{\rho} + gh_2 + \frac{1}{2} v_2^2$$

In general, $\frac{P}{\rho} + gh + \frac{v^2}{2} = \text{constant}$. This is Bernoulli's theorem.



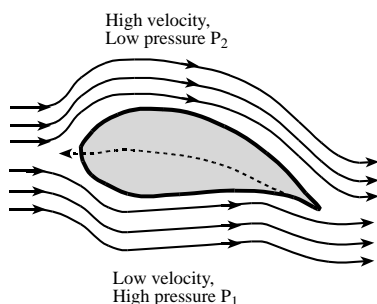
[Fig. 3.22, Bernoulli's theorem]

3.25 Applications of Bernoulli's Theorem

There are many applications of Bernoulli's theorem. Some of them are explained as below.

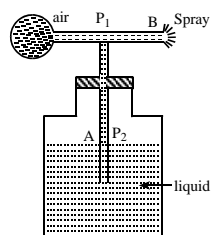
- 1. Lift on an aeroplane:** The shape of the aeroplane wings (i.e., aerofoil) is slightly convex upward and concave downward. Therefore, the speed of air above the wings becomes more than below the wings. So, the pressure below the wings is more than that above the wings. Due to this difference in pressure, a vertical lift acts on the aeroplane. When this lift is sufficient to overcome the

gravitational pull on the aeroplane, it is lifted up. The figure 3.23 shows the lift on an aeroplane wing.



[Fig. 3.23, Lift on an aeroplane wing]

2. **Atomizer:** Atomizer works on the Bernoulli's principle. The schematic diagram of atomizer is shown in figure 3.24. When the rubber balloon is pressed, the air rushes out of the horizontal tube decreasing the pressure P_2 which is less than the atmospheric pressure P_1 in the container. As a result, the liquid rises up in the vertical tube A. When it collides with the high speed of air in tube B, it breaks up into a fine spray.



[Fig. 3.24, Atomizer]

3. **Venturi-meter:** It is a device used for measuring the rate of flow of liquid through pipes. It is based on Bernoulli's theorem.

It consists of horizontal tube (pipe) ABC whose middle part B is narrow. Two vertical tubes E and F are joined at parts A and B as shown in figure 3.25. Both vertical tubes are used to measure pressure of liquids at positions A and B respectively. When liquid flows in tube ABC, then by equation of continuity, the velocity of liquid flow is greater at narrow part B than to that at wide part A. Therefore, by Bernoulli's theorem, $P + \frac{1}{2} \rho v^2 = \text{constant}$, the pressure at narrow part B is less than that at wide part A. This pressure difference can be found by noting the reading of liquid levels in vertical tubes E and F. Suppose, flow of ideal liquid in the tube is streamlines, A_1 and A_2 are area of cross-sections and v_1 and v_2 are the velocities of flow of liquid. P_1 and P_2 are the pressures at positions A and B of tube. As the tube is horizontal, by Bernoulli's theorem,

$P + \frac{1}{2} \rho v^2 = \text{constant}$, the pressure at narrow part B is less than that at wide part A. This pressure difference can be found by noting the reading of liquid

levels in vertical tubes E and F. Suppose, flow of ideal liquid in the tube is streamlines, A_1 and A_2 are area of cross-sections and v_1 and v_2 are the velocities of flow of liquid. P_1 and P_2 are the pressures at positions A and B of tube. As the tube is horizontal, by Bernoulli's theorem,

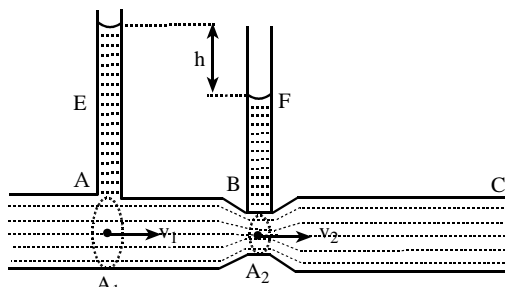
$$\frac{P_1}{\rho} + \frac{v_1^2}{2} = \frac{P_2}{\rho} + \frac{v_2^2}{2}$$

$$\text{or, } P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$\text{or, } \rho gh = \frac{1}{2} \rho \left(\frac{v_2^2}{A_2^2} - \frac{v_1^2}{A_1^2} \right)$$

$$\text{or, } gh = \frac{1}{2} \left(\frac{v_2^2}{A_2^2} - \frac{v_1^2}{A_1^2} \right) = \frac{v^2 (A_1^2 - A_2^2)}{2A_1^2 A_2^2}$$

$$\text{or, } v^2 = \frac{2A_1^2 A_2^2 gh}{A_1^2 - A_2^2}$$



[Fig. 3.25, Venturi-meter]

$\therefore v = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$. Hence, the rate of flow of liquid can be calculated by measuring h .

Boost for Objectives

- For solid body, density of body is equal to density of its substance, (since, $V_{\text{body}} = V_{\text{substance}}$)
- For a hollow body, density of the body is less than density of its material. (since, $V_{\text{body}} > V_{\text{material}}$)
- With increase in pressure, density of substance **increases** due to decrease in volume.
- Relative density of a substance is defined as the ratio of its density to density of water at 4°C.
- Relative density is dimensionless and unitless.
- The liquid pressure is defined as force per unit area. i.e., $P = \frac{F}{A}$
- The pressure of liquid of density ρ at a point at depth h below free surface of liquid is given by $P = h\rho g$
- If P_0 is the atmospheric pressure total pressure at depth h is $P' = P_0 + P = P_0 + h\rho g$
- Liquid pressure depends on h , ρ & g and independent of amount of liquid, shape of vessel and area of consideration.
- Pressure acts equally in all possible directions.
- Liquid pressure is always perpendicular to surface area.
- Upthrust is independent of all factors of body such as its mass, density, size, shape except the volume of body inside the fluid.
- During free fall of vessel containing liquid upthrust is zero.
- The Buoyant force acts at the Centre of Buoyancy which is the Centre of Gravity of the liquid displaced by the body when immersed in the liquid.
- Metacentre is a point where the vertical line passing through the centre of Buoyancy intersects the central line.
- Relative density of a body which is the ratio of density of that body to density of water at 4°C is also equal to the ratio of actual weight of the body to loss in weight when immersed fully in water.
i.e. $R.D. = \frac{\rho_{\text{body}}}{\rho_{\text{water at 4°C}}} = \frac{\text{Weight of body in air}}{\text{Weight of water displaced}} \therefore R.D. = \frac{W_0}{W_0 - W}$
- When a body of density ρ and volume V is immersed in a liquid of density σ , then (i) Weight of the body $= \rho Vg$ acts vertically downward from C.G. (ii) Upthrust $= \sigma Vg$ acts vertically upward from C.G.
- When $\rho > \sigma$, then weight (W) > upthrust (U) so body will sink inside the liquid.
- If $\rho = \sigma$ then $W = U$, body will just float or just sink in the liquid with its entire volume just under the surface of liquid. If the body in such condition is pressed down and released, then it will sink because of weight of additional liquid that comes on its surface.
- A floating body will be in translational equilibrium position if C.G. of the body and C.B. of the upthrust lies in same vertical plane.
- The floating body will be in stable equilibrium when the position of metacentre lies below the centre of gravity of the body. This is found in a case of heavy topped body. It is due to this reason passengers are not allowed to stand on a moving boat.
- The floating body will be in a neutral equilibrium when the position of metacentre coincides with the centre of gravity of the body.
- A vessel containing water is in equilibrium on a beam balance. When a man puts his finger into the water without touching the bottom or the sides of the vessel, the scale pan on which the vessel stands will sink.
- A boy carrying a bucket of water in one hand and a piece of plaster piece to the bucket in which it floats, the boy will carry same load as before.
- Same mass of cotton and iron are weighed in vacuum then both will weight same.
- A piece of ice is floating in a jar containing water. When ice melts, the temperature of water falls from 25°C to 4°C, then the level of water falls.
- A piece of ice is floating in a jar containing water. When ice melts, the temperature of water falls from 4°C to 2°C, then level of water rises.
- A wooden rod can't float vertically in pond water as the metacentre lies below CG.
- With increase in temperature, surface tension generally decreases.
- In case of molten cadmium and copper, increase in temperature increases the surface tension.

- ☞ At critical temperature, surface tension becomes zero.
- ☞ Surface tension of liquid increases when highly soluble impurities (like NaCl) are added due to increase in intermolecular force.
- ☞ But when slightly soluble impurities (like soap, camphor, phenol, etc.) are added, surface tension decreases.
- ☞ Angle of contact is the angle between tangent drawn on liquid surface and solid surface inside liquid at the point of contact.
- ☞ Angle of contact increases if highly soluble impurity is added to water
- ☞ Angle of contact decreases if sparingly soluble impurity is added.
- ☞ Angle of contact decreases on increasing temperature.
- ☞ When two soap bubbles of radii r_1 & r_2 coalesce in vacuum under isothermal condition, energy gets conserved and hence surface area remains unchanged so radius R of big bubble is: $R = \sqrt{r_1^2 + r_2^2}$
- ☞ When height h_1 of capillary tube above surface of liquid is insufficient than actual height h of liquid column, the liquid doesn't overflow. Rather, the liquid rises to entire length of the tube and radius of curvature of liquid increases without changing the nature. Here, $h_1 R = hr$ holds true, where, R = radius of curvature of liquid & r is radius of capillary tube.
- ☞ When several bubble of same liquid get coalesced in vacuum isothermally to form a big bubble, then radius of big bubble is given by $R^2 = r_1^2 + r_2^2 + \dots + r_n^2 \Rightarrow R = N^{1/2}r$ if $r_1 = r_2 = \dots = r$
- ☞ When two bubbles coalesce, energy is released.
- ☞ Angle of contact for pure water and glass is zero.
- ☞ Surface tension of liquid is zero at boiling point.
- ☞ The height up to which water will rise in a capillary tube will be minimum when water temperature is 4°C
- ☞ Due to surface tension, liquid drops assume spherical surface to have minimum surface area.
- ☞ Droplets of a liquid are usually more spherical in shape than large drop of same liquid because force of surface tension predominates the force of gravity.
- ☞ Materials used for water proofing increase both surface tension and angle of contact.
- ☞ Angle of contact varies between $0^\circ - 180^\circ$.
- ☞ Surface tension, elasticity and viscosity arise due to intermolecular cohesive force.
- ☞ The height of liquid column in a capillary tube on the moon is six times that on earth.
- ☞ Potential energy of a molecule on the surface of liquid is greater as compared to one inside the liquid.
- ☞ Rise of oil in wick of lamp is due to capillarity.
- ☞ The radius of soap bubble increases when positive or negative charge is given to the bubble.
- ☞ The maximum velocity of liquid flowing through a tube upto which the flow is streamline is called critical velocity.
- ☞ Reynold's number indicates the nature of flow (i.e., streamline or turbulent).
- ☞ For an ideal liquid flowing through a non-uniform tube, under streamlined condition, the mass flowing per sec is same at each cross-section.
- ☞ Clearly, greater the cross-section, velocity of flowing liquid will be smaller & vice-versa. Due to this reason. (i) Deep water appears to be still. (ii) Falling stream water becomes narrower. (iii) Fine jet is ejected by making cross-section narrower.
- ☞ Equation of continuity is based on conservation of mass.
- ☞ Equation of continuity also refers that volume of liquid flowing per second through any cross-section is constant.
- ☞ Equation of continuity is applied only in case of ideal liquid (non-viscous and incompressible).
- ☞ Bernoulli's theorem is based on principle of conservation of mechanical energy.
- ☞ Pressure decreases when a liquid flows from broader to narrower portion of pipe.
- ☞ Viscous force is due to electromagnetic interaction.
- ☞ Viscosity of liquids decreases with increase of temperature, but viscosity of gases increases with increase of temperature.
- ☞ Viscosity of liquid increases with increase in density while viscosity of gases decreases with increase in density.
- ☞ Viscosity of liquids is due to cohesive force.
- ☞ Viscosity of gases is due to transfer of momentum between gas molecules.
- ☞ Viscosity of liquid increase with increase in pressure of a certain range of pressure. But at very high and low pressure, viscosity is directly proportional to pressure.
- ☞ The design of aeroplane wings is such to obey the Bernoulli's principle, that upper surface is convex and lower one is concave downwards.

- ☞ Viscosity arises due to intermolecular force which is basically electromagnetic forces.
- ☞ Machine parts are jammed in winter due to increase in viscosity of lubricant.
- ☞ The critical velocity (v_c) for non-viscous liquid is zero.
- ☞ Viscosity of water decreases with increase in pressure while that of other liquid increases with increases in pressure.
- ☞ Viscosity of gases remains constant at high pressure.
- ☞ Viscosity of gases is directly proportional to pressure at low pressure region.
- ☞ Rain drops fall with constant velocity due to viscosity.
- ☞ Clouds float in the sky due to their low density.
- ☞ If viscosity of air is taken into account, orbital velocity of satellite moving close to earth increase till the satellite falls back on the earth.
- ☞ Reynold's number is low for high viscosity, low density and low velocity.
- ☞ Poiseuille's law for liquid can be compared with Ohm's law for current flow where viscosity of liquid is equivalent to resistivity.
- ☞ Reynold number is related to critical velocity.



Short Questions with Answers

1. *In a hot air ballooning, a large balloon is filled with air heated by a gas burner at the bottom, why must the air be heated?* [HSEB 2069]
 - ☞ When the air is heated, the density of air decreases and volume increases. So, it displaces the large amount of air and gets sufficient upthrust to lift it up. Therefore, the air is heated to let the balloon up in the sky.
2. *Steel balls sink in water but they don't sink in mercury. Why?* [HSEB, 2052]
 - ☞ The density of steel is higher than water but smaller than mercury. When steel balls are dipped in water, they sink because upthrust due to water is less than the weight of the steel balls. But steel balls don't sink in mercury because upthrust due to mercury is higher than the weight of the steel balls.
3. *Does a ship sink more in river water or in sea water?*
 - ☞ Since, density of sea water is greater than that of fresh water. So same upthrust is achieved in sea by the displacement of smaller volume than in fresh water. Hence, depth of immersion of a ship is less in sea than in fresh water.
4. *What is centre of buoyancy?*
 - ☞ Centre of buoyancy is the C.G. of displaced liquid by the immersion of a body inside the liquid. The Upthrust (force of buoyancy) acts upward through centre of buoyancy on immersed body.
5. *Why does ice float in water?* [HSEB 2050]
 - ☞ For a body to float, the density of body should be less than the density of liquid. Here, the density of ice is less than the density of water. So, ice floats in water.
6. *It is difficult to stop bleeding from a cut in human body at high altitudes, why?*
 - ☞ The atmospheric pressure is low at high altitudes. Due to greater pressure difference between atmospheric pressure and blood pressure, it is difficult to stop bleeding from a cut in human body.
7. *An ice cube floats in a glass of water. When the ice melts, will the water level in the glass rise, fall or remain unchanged?*
 - ☞ The water level remains unchanged. The ice cube displaces a weight of water equal to its own weight. When the ice cube melts, the volume of water produced equals the volume of water, it displaced when frozen.
8. *A helium filled balloon rises to a certain height and then halts. Why?* [HSEB 2060]
 - ☞ Since, the density of helium gas is lower than that of air near the ground. The weight of the balloon will be smaller than the upthrust and the balloon rises to a certain height. It halts at a certain height when upthrust is equal to its weight (where the density of air and helium becomes same).
9. *A cork is floating in water. What is the apparent weight of the cork?* [HSEB 2061]
 - ☞ A body can float in water if the weight of body is equal to upthrust acting on it. The centre of gravity of the body and the centre of buoyancy of displaced water lie in the same line. Thus, the cork becomes weightless (apparent weight becomes zero).
10. *Why is the bottom of a ship made heavy?* [HSEB 2051, 2060]
 - ☞ For the stable equilibrium, the meta-centre must lie

above the centre of gravity for a floating body. When the bottom of the ship is made heavy, the centre of gravity lies lower than meta-centre.

11. Does the Archimedes' principle hold in vessel in free fall?

- ✗ No, the vessel in free fall is in a state of weightlessness i.e., the apparent value of g is zero and hence, the buoyant force does not exist. So, Archimedes principle does not hold good.

12. What is cohesive force?

- ✗ The force of attraction between the molecules of the same substance is called force of cohesion or cohesive force. It is maximum in solids, lesser in liquids and the least in gases.

13. What is adhesive force? [HSEB 2069]

- ✗ The force of attraction between the molecules of the different substances is called the force of adhesion or adhesive force. Force of adhesion is different for different substances. For example, gum has greater adhesive force for a solid surface than water

14. Can you decide whether a liquid will rise or get depressed in a capillary tube by observing the shape of the liquid meniscus?

- ✗ Yes, the level of liquid rises if the shape of meniscus is convex and falls if the shape of the liquid surface is concave in a capillary tube.

15. We use towels to dry our body after bath why? [HSEB 2062]

- ✗ After a bath, there are water drops on our body. By using towels, water is soaked from our body due to capillary action. So, we use towel to dry our body.

16. What do you mean by capillarity?

- ✗ When a capillary tube is dipped in water, water level in tube rises and in mercury, the level is depressed below that of the outside level. This phenomenon is known as capillarity.

17. Define viscosity.

- ✗ The property of fluid by virtue of which it opposes the relative motion of different layers of fluid is called viscosity. It is measured in terms of coefficient of viscosity (η) as given by Newton's formula:

$$F = -\eta A \frac{dv}{dx} \text{ It SI unit is } \text{Ns m}^{-2} \text{ and its dimension is } [\text{ML}^{-1}\text{T}^{-1}].$$

18. Does viscosity depend on temperature?

- ✗ Yes. The viscosity of a liquid varies inversely with its temperature.

19. Machine parts are jammed in cold day. Why? [HSEB 2057]

- ✗ The viscosity of a liquid varies inversely with its temperature. As the temperature decreases in winter,

the coefficient of viscosity of the lubricant increases. Due to the increase in viscosity of lubricant, the machine parts are jammed in water.

20. Why does hotter liquid move faster than cold liquids?

- ✗ Moving of liquids depend on their viscosity. At lower temperature, molecules cannot move fast and so they have high viscosity. So, the liquid themselves cannot move faster. At high temperature however, the viscosity decreases and the molecules move fast, which result in lower velocities. So, the liquids themselves start to move faster.

21. State strokes law. [HSEB 2054]

- ✗ **Stroke's law:** The viscous force F acting on a small sphere of radius r , when it falls through a viscous fluid, depends upon coefficient of viscosity (η) of the fluid, velocity (v) of the sphere and radius (r). It is given by, $F = 6\pi\eta rv$.

22. What is laminar flow of fluid? [HSEB 2052]

- ✗ The flow of a liquid such that the velocity of each layer of the liquid is parallel to the axis of the pipe through which the liquid is flowing is called the laminar flow. Laminar flow makes the velocity of liquids low. At certain velocities called critical velocity depending upon the nature of the liquid, laminar flow changes into turbulent motion.

23. Why do ships and motor boats have sharp edges on the front?

- ✗ Viscous force experienced by a body when moving through a fluid is minimum for a pin pointed shape. Such bodies can acquire maximum possible velocity, through a fluid, than any other shape. Therefore, high speed objects eg. rockets, aeroplanes, submarines, racing cars, missiles have sharp edges on the front.

24. A flask containing glycerin and the other containing water are stirred vigorously and placed on the table. In which flask will the liquid come to rest fast? Why?

- ✗ The liquid, which has got large viscosity, will come to rest faster. Here, the viscosity of glycerin is larger than that of the water, hence glycerin comes to rest faster than water.

25. Why is it easier to lift a body in water than in air?

[HSEB 2059]

- ✗ Upthrust provided by the fluid supports the object to lift the body in it. As we know, upthrust is equal to the weight of fluid displaced, i.e., $U = V\rho g$, where, V is volume of fluid displaced, ρ is density of fluid. Although, the displaced volumes of a body in water and in air are equal, the density of water is greater than the air. Therefore, water provides more upthrust to the body

than the air which makes easier to lift a body in water than in air.

26. Why an air bubble in water rises from bottom top and grows in size?

- ✎ The fluid pressure increases with depth, we have, $P = h\rho g$ and the fluids move from higher pressure to lower pressure. Since, pressure at the top is lesser than that at the bottom; the air bubble will rise from bottom to top. When the bubble moves from higher pressure to lower pressure, its volume increases. According to Boyle's law, $PV = \text{constant}$. It means that, radius increases gradually from bottom to the top.

27. State the laws of floatation. [HSEB 2055]

- ✎ **Law of floatation:** For a body floating freely in a liquid, of the laws of floatation are
1. The weight of the body is equal to the weight of the displaced liquid.
 2. The centre of gravity of the body and the centre of gravity of the displaced liquid are in same vertical line

28. A piece of ice is floating in water. Will the water level rise if the ice melts completely? Explain. [HSEB 2068]

- ✎ No, The water level will remain same if piece of ice melts completely in a container of water. Let V' be the volume of ice cube inside water and V the whole volume of ice. For floating body,
- $$\text{weight of ice} = \text{upthrust}$$
- $$m_i \times g = V' \times \sigma_w g$$
- $$\text{or, } V \times \rho_i = V' \sigma_w$$
- $$\text{or, } V' = V \times \frac{\rho_i}{\sigma_w}$$

When whole ice melts,

$$\begin{aligned} \text{volume of water obtained} &= \frac{\text{mass of water}}{\text{density of water}} \\ &= \frac{\text{mass of ice}}{\text{density of water}} = \frac{V \times \rho_i}{\sigma_w} \end{aligned}$$

Since, the volume of water obtained from melted ice is equal to the volume of ice inside water level, so water level remains the same.

29. How will you make difference between density and specific gravity of a body? [HSEB 2062]

- ✎ The differences between density and specific gravity are:

Density	Specific Gravity
a. The quantity of matter (mass) per unit volume is called density of a substance.	a. The mass of any volume of it as compared to the mass of the same volume of water at 4°C is called specific gravity of a substance.
b. It is absolute value.	b. It is the relative density.
c. It's unit is Kg/m^3 in SI and gm/cm^3 in CGS.	c. It is unit less quantity.

30. Lead has a greater density than iron, and both are denser than water. Is the buoyant force on a lead object greater than, less than, or equal to the buoyant force on an iron object of the same volume? [HSEB 2054]

- ✎ We know that,

Upthrust = Weight of water (fluid) displaced

or, $U = \text{mass of water displaced (m)} \times g$

or, $Ug = \text{volume of water displaced (V)} \times \text{density of}$

$$\text{water } (\rho) \times g \left(\because \rho = \frac{m}{V} \right)$$

or, $U = V\rho g \quad \dots (1)$

If lead and iron objects have equal volume, they displace equal volume of water. Hence, the equation (1) indicates that buoyant force on the lead object is same as that on the iron object of same volume, whatever be, their densities.

31. The purity of the gold can be tested by weighing it in air and water. How? [HSEB 2070]

- ✎ The purity of gold can be tested by determining its specific gravity (density cgs unit). Although, the mass and volume of a substance varies, the specific gravity remains constant at certain temperature. It is calculated from the formula for gold.

$$\text{Specific gravity} = \frac{\text{weight of gold in air}}{\text{weight of gold in water}} \times \text{Sp. gravity of water at } t^\circ\text{C}.$$

The standard value of specific gravity of pure gold is 19.3. If the measure value of specific gravity is deviated from the standard value of specific gravity, the gold is impure. If specific gravity of gold is observed ≈ 19.3 , the gold is pure.

32. A piece of iron sinks in water, but a ship made of iron floats in water, why?

- ✎ The weight of water displaced by iron piece is less than its own weight. So, it does not get proper amount of upthrust, to hold on the surface of water and hence, it sinks. But, the ship displaces water more than its own weight, so it gets sufficient amount of upthrust to hold on the surface of water and hence, it floats.

33. A body floats in a liquid contained in a beaker. The whole system falls under gravity. What is upthrust on the body due to liquid? [HSEB 2063]

- ✎ A body can float in water if the weight of body is equal to upthrust acting on it. But when the whole system falls under gravity, $a = g$ and we have
- $$\text{Weight of the body} - \text{upthrust} = ma$$
- $$\text{or, } mg - \text{upthrust} = ma$$
- $$\text{or, } \text{upthrust} = mg - ma$$
- $$\text{or, } \text{upthrust} = mg - mg = 0$$
- Therefore, upthrust = 0, hence, upthrust on the body falling freely under gravity is zero.

34. Explain why rain drops are spherical in shape. or why are liquid drops spherical in shape? explain.

[HSEB 2053, 2056, NEB 2074]

- ✎ A drop of liquid in equilibrium is acted by gravitational force and the forces due to surface tension. For a small drop, the gravitational potential energy is smaller than potential energy due to surface tension. Hence, to keep equilibrium, the liquid tends to contract so that its surface area should be minimum and the drop becomes spherical.

35. Why mercury does not wet the glass tube? [HSEB 2058]

- ✎ When mercury comes in contact with glass tube, it forms a convex meniscus. Those liquids, which produce convex meniscus when come in contact with solid, do not wet the solid. So, mercury does not wet the glass tube. Also, mercury has high cohesive force than adhesive force. Due to this, it has obtuse angle of contact with the substance i.e., with the glass. Because of this high cohesive force, the molecules of mercury are bound to themselves and do not wet the glass.

36. Why is soap solution a better cleansing agent than ordinary water? [HSEB 2066]

- ✎ The surface tension of soap solution is less than the ordinary water due to which the water reaches to every region and cleans them. If the surface tension of water is high, the water does not enter deep inside part of cloths and cannot remove the dust particle due to less surface tension the solution is a better cleansing agent than ordinary water.

37. Why are small drops of mercury spherical and bigger drops oval in shape? [HSEB 2060]

- ✎ In case of small drops of mercury, the gravitational potential energy is negligible in comparison to the potential energy due to surface tension. Consequently, to keep the drop in equilibrium, the mercury drop surface tends to contract so that its surface area will be minimum for a sphere and the drops will be spherical. But, in case of bigger drops of mercury, the potential energy due to gravity is predominant over the potential energy due to the surface tension. Consequently, to keep equilibrium the mercury drop tends to assume minimum potential energy as possible so as to make the gravitational potential energy as small as possible, the drop comes in oval shape and lowers the centre of gravity.

38. Hairs of a brush spread out when it is dipped in water and cling together as soon as it is taken out of water. Explain. [HSEB 2061]

- ✎ The free surface of a liquid acts as stretched membrane due to surface tension. Hairs of a brush cling together

when it is taken out of water as the free surface of films try to contract due to surface tension. There is no surface tension inside water and hence, hairs of the brush spread out there.

39. Why the antiseptics used for cuts and wounds in human flesh have low surface tension? [HSEB 2069]

- ✎ If the antiseptics used for cut and wounds in human flesh have low surface tension, they reach to each & every parts of wounds and make relief from pain as well as cure fast.

40. Soaps and detergents are used to clean the clothes, why? [HSEB 2066]

- ✎ Soaps and detergents are the substances which can reduce the surface tension of water. When surface tension of water decreases, then the water can penetrate easily between the fibres of material of the cloths. Then, the dust particles and stains from the clothes will be washed easily and makes the clothes clean. Hence, to clean cloths soaps or detergents are used such substances which reduces surface tension of a liquid are called surfactants.

41. Why there is a split in the nib of a pen? [HSEB 2060]

- ✎ The nib of a pen has a split at the centre to make the flow of ink through it. Because, when splitted, the nib acts as a capillary tube. So, ink rises due to capillary, whether pen is kept down or up. Then, during writing ink flows to the paper and marks the paper which makes possible for writing.

42. Small particles of camphor dance on the surface of water, why? [HSEB 2063]

- ✎ If a small particle of camphor is floated upon the surface of clean water, a part of it is gradually dissolved the surface tension of the solution is less than that of clear (pure) water hence, the camphor particles are pulled towards pure water (i.e. towards the region of higher surface tension). Since, camphor is continuously dissolving; the force of surface tension is not uniform all round with the result that the camphor pieces dance haphazardly in different direction on the surface of water.

43. Why does hot soup taste better than cold one?

- ✎ Tongue can detect taste by complex organic reaction with the food. Hot soup helps organic reaction faster and makes the soup tasty. But, cold soup takes heat during reaction ruining the taste of reaction. At higher temperature, surface tension and surface area increases. Increase in surface area spreads the taste at large area as compared to cold. So, the hot soup tastes better than cold one.

44. It is observed that the surface of a liquid in a capillary tube dipped in it is either convex or concave. What may be the reason? Explain. [HSEB 2072]

- ✎ It is observed that the surface of a liquid in a capillary tube dipped in it is either convex or concave. The reason behind this is cohesive force of liquid molecules and adhesive force between liquid molecule and molecules of tube. If cohesive force between liquid molecules is more, the surface is convex and if the adhesive force between liquid and solid molecules is more, then the surface of liquid is concave

45. Why insects can walk on the surface of water?

- ✎ The insects can walk on the surface of water without sinking on the water, due to surface tension of water. Actually leg of the insect sink slightly into the liquid, hence, the weight of the insect i.e., its true weight is less than the buoyant force.

46. How do the leaves of tree get water from ground?

[HSEB 2068]

- ✎ The leaves of tree get water from ground by capillary action. The stem of the body of tree consists of small bores like capillary tube. Since, the roots are deep in contact with the water resource below the ground, water travels through the channels of bores according to the relation $h = \frac{2T \cos \theta}{r \rho g}$. Since, the pores are very fines, r is very small. So, they can rise to leaves

47. Why is it easier to wash clothes in hot water or soap solution or by using detergent?

- ✎ Soap helps in cleaning the clothes is related to the access of water to dirt and other unwanted substances and carry them away. This requires that water does not block itself and other because of its high surface tension. For this, that the surface tension of water be reduced which can be achieved by adding foreign substances as detergents and soap. Besides, the increase of temperature also decreases the surface tension, so water can reach every region and clean them.

48. Dust particles seem to be floating in the air, why?

- ✎ Dust particles have very small mass and very small dimensions as well, which means they have small ' r ' (though they are exactly not spherical). Since the terminal velocity depends on ' r ', as shown by the relation $V_t = \frac{2r^2(\rho - \sigma)g}{9\eta}$, their terminal velocity will be very small. This means they fall at a very small rate, so small that the fall is almost negligible. That's why, they seem to float in the air.

49. Express the dimension of the velocity gradient from the definition of the coefficient of viscosity. [HSEB 2071C]

- ✎ The velocity gradient $\left(\frac{dv}{dx}\right)$ in terms of coefficient of viscosity (η) is given by, $F = \eta A \frac{dv}{dx}$ (in magnitude), where A is area of surface of constant and F is viscous force, Now,
 $\frac{dv}{dx} = \frac{F}{\eta A}$
 Here, dimension of $F = [MLT^{-2}]$
 dimension of $\eta = [ML^{-1}T^{-1}]$
 dimension of $A = [L]^2$
 Then, $\frac{dv}{dx} = \frac{[MLT^{-2}]}{[ML^{-1}T^{-1}][L]^2} = [T^{-1}]$.
 Hence, dimension of $\frac{dv}{dx}$ is $[T^{-1}]$

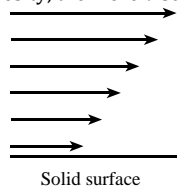
50. When a smooth flowing stream of water comes out of a faucet, it narrows as it falls. Explain why this happens? [HSEB 2066]

- ✎ When a smooth flowing stream of water falls from a certain height, the velocity of water increases as it travels distance. We know from the equation of continuity $A_1V_1 = A_2V_2$ where, A and V are area and velocity respectively. From this relation, if viscosity increases, the area should be decreased. Hence as waterfalls, velocity increases and it becomes narrow.

51. Explain, what is meant by the fact that fluid is viscous?

[HSEB 2053]

- ✎ In fluid motion, there is relative motion between different layers of the fluids. The velocity of the contact with the solid surface is practically zero and it increases as we move towards free surface. Because the velocities of neighboring layers are different, a frictional force occurs between the various layers of a liquid (fluid) when flowing over a solid surface. To maintain relative motion between different layers of the fluid, an external force must be applied to it. In the absence of external force, the flow of liquid (fluid) increases. So, the fluid is said to be viscous. The higher the coefficient of viscosity, the more viscous will be the fluid.

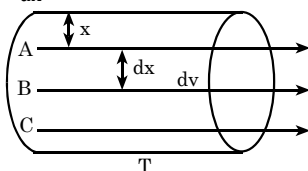


Solid surface

52. Explain with a diagram, the meaning of velocity gradient in the case of liquid flowing in a tube. [HSEB 2072]

- ✎ Let us consider, a tube T in which a liquid is flowing as shown in figure. Let, A, B and C be three layers of liquid in the tube. A layer A is at a distance x from the surface of tube moving at a velocity v . The layer B is at a distance $(x + dx)$ from the surface of tube and moving with a velocity of $(v + dv)$. Hence, the velocity increases an increasing distance. The change in velocity per unit change in distance is velocity gradient. i.e. Velocity

$$\text{gradient} = \frac{dv}{dx}$$



53. Smoke rises in a chimney faster when a breeze is blowing. Use Bernoulli's principle to explain this phenomenon. [HSEB 2072]

- ✎ According to Bernoulli's principle, in the flow of fluid, the pressure is maximum where the velocity is the minimum and vice-versa. The breeze blowing over the chimney creates low pressure there than the pressure inside the chimney. So, the smoke rises fast from high pressure to low pressure. Thus, because of pressure difference, smoke rises fast.

54. Why do small air bubbles rise slowly while big rise rapidly through the liquid?

- ✎ The terminal velocity of an object through a fluid is proportional to the square of its radius. This fact says that terminal velocity of big air bubbles is greater than those of small one. Hence, big bubbles rise rapidly than small one.

55. Why does flag flutter when strong winds are blowing on a day?

- ✎ During strong wind, the velocity of the wind at the top is more than that of air below it. Then, accordingly to the Bernoulli's equation, the pressure of the air at the top decreases. Thus, due to the difference in the pressure between two levels, flow of air takes place and makes the flag to flutter.

56. Distinguish between viscosity and friction.



Viscosity		Friction	
i.	It is opposition between two layers of a fluid.	i.	It is the opposition between two solid surfaces.
ii.	The viscous force is proportional to the area of contact between the two layers of the fluid.	ii.	The force of friction is independent of area of contact between two surfaces.
iii.	The viscous force depends upon the relative motion as well as the distance between two layers of the fluid.	iii.	The force of friction is independent of the relative motion between two bodies.
iv.	The viscosity of a fluid either decreases or increases with the rise of temperature.	iv.	The force of friction is independent of temperature.

57. Air pressure decreases with increase in altitude. So why is air near the surface not continuously drawn upward toward the lower - pressure regions above?

- ✎ Air pressure decreases with increase in altitude but the air near the earth surface is not continuously drawn upward toward the lower pressure regions above because air molecules are constantly pulled downward due to gravity.

58. Define coefficient of viscosity and poise. [HSEB 2069]

- ✎ The coefficient of viscosity is defined as $F = -\eta A \frac{dv}{dx}$.. (1)

Where A is area of layer of fluid, $\frac{dv}{dx}$ is the velocity gradient and η is a constant of proportionality and is called coefficient of viscosity. The negative sign shows that the direction of viscous force, F is opposite to the direction of motion of the liquid. If $A = 1$ and $\frac{dv}{dx} = 1$, then

from equation (1), we have, $\eta = -F$. Hence, coefficient of viscosity of a liquid is defined as the viscous drag or viscous force acting per unit area of the layer having unit velocity gradient perpendicular to the direction of the flow of the liquid. Poise is the unit of coefficient of viscosity

59. Why do the clouds seem floating in the sky?

- ✎ Clouds are actually not floating, like every other substances, they are also falling. But the way they fall is different. Cloud is basically made of tiny water drops plus dust particles associated with them. However, because of their small size, the terminal velocity attained by them is also very low according to the

relation $v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}$. Such velocity is so less that for an observer down on the earth. Hence, they just seem like floating.

60. Explain in brief how an aeroplane takes off? Use Bernoulli's principle.

- ✎ The wings of planes are designed in very special way. The upper front parts of the wings are highly curved compared to the upper back portions. When the planes with such wings move forward, the air passing from above will travel faster to reach the back compared to the air passing from below. So, according to the Bernoulli's relation, $P + \frac{1}{2}\rho v^2 = \text{constant}$. The pressure above will be lesser than the pressure below. This will create a resultant upward force. If the plane moves fast enough, the force will be large enough to lift the massive plane as well.

61. During certain windstorm, light roofs are blown off. Why? [HSEB 2060]

- ✎ According to Bernoulli's principle, ($P + \frac{1}{2}\rho v^2 = \text{constant}$) the pressure on the top of region is lower than that at bottom region. The inside pressure becomes very much larger than the outside pressure of the roof because the inside air has lower kinetic energy than that of outside air of the roof. This difference of pressure results in an increment of up thrust. As a result, the roof is lifted up and blown off by the wind.

62. Airports at high elevations have longer runways for take offs and landings than do airport at sea level, why? [HSEB 2071S]

- ✎ From Bernoulli's Theorem, $P + \frac{1}{2}\rho v^2 = \text{constant}$. Where P = pressure, ρ density and v = velocity.

Airport at high elevation has low air pressure than the airport at low elevation and the velocity of airplane is greater in high elevation airport than low elevation airport. Hence, the airports at high elevations have longer runways for take offs and landings than do airports at sea level.

63. An aeroplane requires a long run on the ground before taking off, why?

- ✎ The lifting of a plane requires that the upward force to be created on its wings should be more than its weight. For this purpose, the wings of planes have to be designed in very special way. The upper front parts of the wings are highly curved compared to the upper back portions. When the planes with such wings move forward, the air passing from above will travel faster to reach the back compared to the air passing from below.

So, according to the Bernoulli's relation, $P + \frac{1}{2}\rho v^2 = \text{constant}$, the pressure above will be lesser than the pressure below. This will create a resultant upward force. If the plane moves fast enough, the force will be large enough to lift the massive plane as well. To achieve this, the run up should be long.

64. Explain why a suction effect is experienced by a person standing close to the platform at a station when a fast train passes. [HSEB 2067, 2072]

- ✎ When a fast moving train passes by a person standing nearby its track, the air between the person and the train moves with a high speed. According to the Bernoulli formula $P + \frac{1}{2}\rho v^2 = \text{constant}$, the air pressure becomes low near him/her compared to the pressure at the other regions including behind the person. So the air moves from back side of the person to front side pushing him/her on the way.



Worked Out Examples

1. Piece of gold aluminum alloy weight 100 gm in air and 80 gm in water. What is the weight of gold in the alloy if the relative density of gold is 19.3 and that of aluminum. [HSEB 2055]

Solution:

Let, x gm be the weight of gold in the alloy, then the weight of aluminum in the alloy = $(100 - x)$ gm
Weight of water displaced by the alloy = $100 - 80$
= 20 gm

Therefore, volume of the alloy = $\frac{20}{1} = 20 \text{ cm}^3$

Volume of aluminum in the alloy = $\frac{100 - x}{2.5}$

$$\frac{100 - x}{19.3} + \frac{100 - x}{2.5} = 20$$

$$0.052x + 40 - 0.4x = 20$$

$$\text{or, } 0.348x = -40 + 20$$

$$\text{or, } x = \frac{-20}{-0.348}$$

$$\therefore x = 57.5 \text{ gm.}$$

Hence, the weight of gold in the alloy is 57.59 gm.

2. A boy can lift a maximum load of 250N of water. How many liters of mercury (density 13600 kg m^{-3}) can be lift if contained in an identical vessel? [HSEB 2066]

Solution:

The maximum load (F) = 250N
 density of mercury (ρ) = 13600 kg m^{-3}
 volume of mercury (V) = ?

We have,

$$F = mg = V \cdot \rho \cdot g$$

$$\text{or, } 250 = 13600 \cdot V \cdot 10$$

$$\text{or, } V = 1.8838 \times 10^{-3} \text{ m}^3$$

$$\text{or, } V = 1.838 \text{ litres}$$

Hence, 1.838 litres of mercury a boy can lift.

3. A string supports a solid iron object of mass 200gm totally immersed in a liquid of density 800 kg m^{-3} . Then density of iron is 8000 kg m^{-3} . Calculate the tension in the string. [HSEB 2057, 2062, 2065]

Solution:

Given, Mass of an iron object (m) = 200gm = 0.2kg

Density of iron (ρ_1) = 8000 kg m^{-3}

Density of liquid (ρ_2) = 800 kg m^{-3}

When a solid iron object is immersed completely in a liquid, total upward force is equal to the total downward force in equilibrium.

$$\therefore T + U = mg$$

Where, T is the tension in the string U is the upthrust due to the liquid and mg is the weight of the iron object.

$$T = mg - \text{weight of liquid displaced} \\ = mg - (\text{mass of liquid displaced}) \times g$$

$$= mg - \rho_2 \cdot V \cdot g = mg - \rho_2 \frac{m}{\rho_1} g$$

$$= 0.2 \times 10 - 800 \times \frac{0.2}{8000} \times 10 = 1.8 \text{ N}$$

Hence, tension in the string $T = 1.8 \text{ N}$.

4. A wooden block of mass 10 kg is floating in water keeping $\frac{1}{3}$ of its volume in air. Find the minimum mass to be placed on the wooden block so that the block is completely immersed in water. [HSEB 2067]

Solution:

Here, wooden block mass (m) = 10 kg

$$\text{Volume in air } (V_a) = \frac{1}{3} V$$

$$\text{Volume in water } (V_w) = 1 - \frac{1}{3} V = \frac{2}{3} V$$

For floating body,

weight of body = weight of displaced liquid

$$\text{or, } V \times \rho = \frac{2}{3} V \times 1$$

$$\therefore \rho = \frac{2}{3} = 0.667 \text{ gm cm}^{-3} = 666.67 \text{ kg/m}^3$$

$$\text{Now, volume of body } (v) = \frac{m}{\rho} = \frac{10}{\frac{2000}{3}} = \frac{30}{2000} = 0.015 \text{ m}^3$$

$$\text{Volume of body to be dipped } (V') = \frac{V}{3} = \frac{0.015}{3} = 0.005 \text{ m}^3$$

Therefore, minimum mass to be placed (m') = $V' \rho$

Where, ρ is the density of displaced liquid

$$= 0.005 \times 1000 \text{ kg} = 5 \text{ kg.}$$

Hence, the minimum mass to be placed on the wooden block is 5 kg.

5. An iceberg having volume 2200 cc floats on sea water (density 1.03 gm cc) with a portion 234 cc above the surface, calculate the density of ice. [HSEB 2056]

Solution:

Given, volume of iceberg = 2200 cc;

Portion of iceberg above water = 234 cc

Density of sea water = 1.03 gm/cc ;

Density of ice = ?

We proceed,

$$\text{Volume of ice in water} = 2200 - 234 = 1966 \text{ cc}$$

$$\text{Upthrust} = \text{wt. of displaced water}$$

$$= \text{volume of immerse ice} \times \text{density of water} \times \text{acceleration due to gravity.}$$

$$\text{Thus, mass of displaced water} = 1966 \times 1.03 = 2024.98 \text{ gm}$$

From principle of floating,

$$\text{wt. of floating body} = \text{wt. of displaced liquid.}$$

$$\text{So, mass of ice} = \text{mass of displaced water} = 2024.98 \text{ gm}$$

$$\text{Density of ice} = \frac{2024.98}{2200} = 0.9204 \text{ gm/cc.}$$

Thus density of ice is 0.9204 gm/cc

6. What is the area of the smallest block of ice, 25cm thick that will just support a man weight 80 kg? The specific gravity of ice is 0.917 and it is floating in fresh water. [HSEB 2052]

Solution:

Let, 'A' be the area of the block of ice.

Weight of the man and the block of ice

$$= 80 \times 1000 \text{ gm} + (25 \times A) \times 0.917 \text{ gm}$$

$$\text{Weight of water displaced} = \text{volume} \times \text{density} \\ = A \times 25 \times 1 \text{ gm}$$

Therefore, the weight of the man and the block = weight of water displaced

$$8000 + A \times 25 \times 0.917 = A \times 25$$

$$\text{or, } 8000 = A \times 25 (1 - 0.917)$$

$$\therefore A = 38554.21 \text{ cm}^2 = 3.85 \text{ m}^2$$

Hence, the required area is 3.85 m^2 .

7. The density of the ice is 971 kgm^{-3} and the approximate density of the sea-water in which an iceberg floats is 1025 kgm^{-3} . What fraction of the iceberg is beneath the water surface? [HSEB 2051]

Solution:

Given, Density of ice (ρ_i) = 971 kgm^{-3}
 Density of sea-water (ρ_w) = 1025 kgm^{-3}
 Fraction of ice beneath the water surface = ?
 From the principle of flotation,
 Weight of ice = weight of water displaced
 So, $V_i \rho_i g = V_w \rho_w g$
 or, $\frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{971}{1025} = 0.95$.
 Hence, the fraction of the iceberg beneath the water surface is 0.95.

8. A string supports a solid iron object of mass 180 gm totally immersed in a liquid of density 800 kgm^{-3} . The tension of the iron is 8000 kgm^{-3} . Calculate the tension in the string. [HSEB 2054]

Solution:

Given, Mass of solid iron (m_i) = $180 \text{ gm} = 0.180 \text{ kg}$
 Density of liquid (ρ_l) = 800 kgm^{-3}
 Density of iron (ρ_i) = 8000 kgm^{-3}
 Tension on the string (T) = ?
 We have, the tension on the string
 $T = W_i - U = m_i g - m' g$

$$= 0.18 \times 9.8 - \rho_l \times V_i \times g \quad \left[\because V_i = V_l = \frac{M_i}{\rho_i} \right]$$

$$= 0.180 \times 10 - \rho_l \left(\frac{m_i}{\rho_i} \right) g$$

$$= 0.180 \times 10 - 800 \left(\frac{0.180}{8000} \right) \times 10$$

 $\therefore T = 1.62 \text{ N}$
 Hence, the tension in the string is 1.62 N .

9. An iceberg having a volume of 2060 cc floats in sea water of density 1030 kgm^{-3} with a portion of 224 cc above the surface. Calculate the density of ice. [HSEB 2056, NEB 2074]

Solution:

Given, Volume of iceberg (V_i) = $2060 \text{ cc} = 2060 \times 10^{-6} \text{ m}^3$
 Volume of iceberg above the surface of water,
 $(V_1) = 224 \text{ cc} = 224 \times 10^{-6} \text{ m}^3$
 Volume of iceberg below the surface of water,
 $(V') = V - V_1 = (2060 - 224) \text{ cc} = 1836 \times 10^{-6} \text{ m}^3$
 Density of water (ρ_w) = 1030 kg m^{-3}
 Density of iceberg (ρ_i) = ?
 By the principle of flotation,
 The weight of iceberg = The weight of water displaced.
 or, $m g = m' g$

or, $\rho_i V_i g = \rho_w V' g$
 or, $\rho_i \times 2060 \times 10^{-6} \times g = 1030 \times 1836 \times 10^{-6} \times g$
 or, $\rho_i = 1030 \times \frac{1836}{2060}$
 $\therefore \rho_i = 918 \text{ kg m}^{-3}$
 Hence, the density of ice is 918 kgm^{-3}

10. A string supports a solid iron of mass 200 kg totally immersed in a liquid of density 900 kgm^{-3} . Calculate the tension in the string if the density of the iron is 8000 kgm^{-3} . [HSEB 2057]

Solution:

Given, mass of solid iron (m) = $200 \text{ g} = 0.2 \text{ g}$
 Density of liquid (ρ_l) = 900 kg m^{-3}
 Tension in the string (T) = ?
 The tension on the string (T) = $W - U = m g - m' g$

$$= 0.2 \times 9.8 - \rho_l V g = 0.2 \times 9.8 \times \rho_l \left(\frac{m}{\rho} \right) g$$

$$= 0.2 \times 9.8 - 900 \times \left(\frac{0.2}{8000} \right) \times 9.8 = 1.74 \text{ N}$$

 Hence, the tension in the string is 1.74 N

11. A geologist finds a moon rock, whose mass is 7.2 kg has an apparent mass 5.88 kg when submerged in water. What is the density of the rock? [HSEB 2066]

Solution:

Given, the mass of a moon rock (m_r) = 7.2 kg
 The apparent mass (m_a) = 5.88 kg (When submerged in water)
 Density of rock (ρ) = ?
 According to Archimedes's Principle we have,
 Apparent wt = Real wt. - Upthrust
 or, $W_a = W - U$
 or, $m_a g = m_r g - m' g$
 or, $m_a = m_r - m'$
 or, $m_a = m_r - V \rho_w$
 or, $V \rho_w = m_r - m_a$
 or, $V = \frac{m_r - m_a}{\rho_w} = \frac{7.2 - 5.88}{1000}$
 or, $V = 1.32 \times 10^{-3} \text{ m}^3$

Density of the rock, (ρ_r) = $\frac{m_r}{V} = \frac{7.2}{1.32 \times 10^{-3}} = 5454.54 \text{ kgm}^{-3}$

Hence, the require density of rock is 5454.54 kgm^{-3} .

12. Calculate amount of energy needed to break a drop of water of diameter 0.01 m into 10^6 droplets of equal sizes. The surface tension of water is 0.072 Nm^{-1} . [HSEB 2066]

Solution:

Here, total surface energy before E_{int} is given by,
 $E_{\text{int}} = T \cdot 4\pi R^2$ [since, $E = T \cdot A$] = $T \cdot \pi D^2$

$$= 0.072 \times 3.14 \times (0.01)^2$$

$$\therefore E_{\text{int}} = 2.26 \times 10^{-5} \text{ J}$$

Now, let, d be diameter of each drop after it breaks down,

The initial volume of large drop = The total volume of all drops.

$$\frac{\pi D^3}{6} = N \frac{\pi}{6} d^3 \quad [\text{Where, } N = 10^6 \text{ no. of total drop.}]$$

$$\therefore d = 10^{-4} \text{ m}$$

Now, total final energy

$$E_{\text{final}} = N \times \pi d^2 \times T$$

$$= 10^6 \times 3.14 \times (10^{-4})^2 \times 0.072 = 2.26 \times 10^{-3} \text{ J}$$

Energy needed to break,

$$E = E_{\text{final}} - E_{\text{int}} = 2.26 \times 10^{-3} - 2.26 \times 10^{-5}$$

$$\therefore E = 2.23 \times 10^{-3} \text{ J}$$

Hence, required energy is $2.23 \times 10^{-3} \text{ J}$

- 13.** Find the work done required to break up a drop of water of radius $5 \times 10^{-3} \text{ m}$ into eight drops of water, assuming isothermal condition. [HSEB 2071]

Solution:

Given, radius of large drop (R) = $5 \times 10^{-3} \text{ m}$

Number of drops (n) = 8

Work done (w) = ?

Surface Tension of water (T) = $72 \times 10^{-3} \text{ Nm}^{-1}$

Since, mass of water drop remains constant, Thus,

$$\left(\frac{4}{3} \pi R^3\right) \rho = 8 \left(\frac{4}{3} \pi r^3\right) \rho$$

$$\text{or, } R^3 = 8 r^3$$

$$\text{or, } R = 2 r$$

$$\text{or, } r = \frac{R}{2} = \frac{5 \times 10^{-3}}{2} = 2.5 \times 10^{-3} \text{ m}$$

$$\begin{aligned} \text{Area of large drop} &= 4\pi R^2 = 4\pi (5 \times 10^{-3})^2 \\ &= 3.14 \times 10^{-4} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of smaller drops} &= 8 \times 4\pi r^2 \\ &= 8 \times 4\pi \times (2.5 \times 10^{-3})^2 \\ &= 6.28 \times 10^{-4} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Increase in area} &= (6.28 - 3.14) \times 10^{-4} \text{ m}^2 \\ &= 3.14 \times 10^{-4} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Work done} &= \text{Increase in area} \times T \\ &= 3.14 \times 10^{-4} \times 10^{-3} = 2.26 \times 10^{-5} \text{ J} \end{aligned}$$

- 14.** A rectangular plate of dimensions 6 cm by 4cm and thickness 2mm is placed vertical so that its largest side just touches the surface of water. Calculate the downward force on the plate due to surface tension. Given surface tension of water = $7 \times 10^{-2} \text{ Nm}^{-1}$. [HSEB 2060, 2069, 2071]

Solution:

For rectangular plate:

Length (l) = 6cm = $6 \times 10^{-2} \text{ m}$

Breadth (b) = 4cm = $4 \times 10^{-2} \text{ m}$

Thickness (t) = 2mm = $2 \times 10^{-3} \text{ m}$

For water:

Surface tension (T) = $7 \times 10^{-2} \text{ Nm}^{-1}$

Downward force on the plate due to surface tension when its largest side just touches the surface of water (F_d) = ?

$$\text{Mathematically, Surface tension, } T = \frac{F}{L}$$

$$\text{or, } F = T \times L$$

Where, F is force due to surface tension and L is the total length on which surface tension acts.

$$\therefore F_d = F = T \times L \quad \dots (1)$$

When the largest surface of a rectangular plate touches the surface of water, then,

$$L = 2(l + t) = 2(6 \times 10^{-2} + 2 \times 10^{-3})$$

$$\text{or, } L = 0.124 \text{ m}$$

So, from equation (1), we get,

$$F_d = T \times L = 7 \times 10^{-2} \times 0.124$$

$$\therefore F_d = 8.68 \times 10^{-3} \text{ N}$$

Hence, the required downward force when the largest side of rectangular plate just touches the surface of water is $8.68 \times 10^{-3} \text{ N}$.

- 15.** Two spherical rain drops of equal rise are falling vertically through air with a terminal velocity of 0.150 ms^{-1} . What would be the terminal velocity if these two drops were to coalesce to form a larger spherical drop? [HSEB 2065]

Solution:

Given, Velocity of rain drop (V) = 0.15 ms^{-1}

Let, r be the radius of the rain drops.

We know, for rain drops, viscous force $F = \sigma \pi \eta r v$

Weight of each rain drop

$$W = \frac{4}{3} \pi r^3 \rho g \quad (\text{neglecting upthrust})$$

$$F = W$$

$$\text{or, } 6\pi \eta r v = \frac{4}{3} \pi r^3 \rho g \quad \dots (1)$$

Let, R be the radius of the larger drop, then,

$$\frac{4}{3} \pi R^3 = 2 \times \frac{4}{3} \pi r^3$$

$$\text{or, } R = 2^{1/3} r$$

Hence, for large drop.

$$6\pi \eta R V = \frac{4}{3} \pi R^3 \rho g \quad \dots (2)$$

Where, V is the terminal velocity of the larger drop.

Dividing (2) by (1) we get,

$$\frac{R V}{r v} = \left(\frac{R}{r}\right)^3$$

$$\text{or, } \frac{V}{v} = \left(\frac{R}{r}\right)^3$$

$$\text{or, } V = \left(\frac{2^{1/3} r}{r} \right)^2 \times v = 2^{2/3} \times 0.15 = 1.587 \times 0.15$$

$$\therefore V = 0.238 \text{ ms}^{-1}$$

Hence, terminal velocity is 0.238 ms^{-1} .

- 16.** With what terminal velocity will an air bubble 1mm in diameter rise in a liquid of viscosity 150 poise and density 0.9 gm cm^{-3} . [HSEB 2050]

Solution:

For an air bubble:

Diameter (d) = 1 mm

$$\text{Radius (r)} = \frac{d}{2} = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

Terminal velocity (V) = ?

For a liquid:

Coefficient of viscosity (η) = 150 poise = 15 Nsm^{-2}
[Since, 10 Poise = 1 Nsm^{-2}]

Density (σ) = $0.9 \text{ g/cc} = 0.9 \times 10^3 \text{ kg/m}^3$

We know that, when a body falls with terminal velocity then, the terminal velocity acquired by the body (from application of stoke's law) is,

Weight of body = Viscous force + upthrust

$$\text{or, } mg = 6\pi\eta rv + m'g$$

$$\text{or, } \rho Vg = 6\pi\eta rv + \sigma Vg$$

$$\text{or, } \frac{4}{3} \pi r^3 \rho g = 6\pi\eta rv + \frac{4}{3} \pi r^3 \sigma g$$

$$\therefore v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

Neglecting the density of air (ρ) = 0

$$v = -\frac{2r^2\sigma g}{9\eta} = \frac{2 \times (0.5 \times 10^{-3})^2 \times (0.9 \times 10^3) \times 9.8}{9 \times 15}$$

$$\therefore v = -3.27 \times 10^{-5} \text{ ms}^{-1}$$

Hence, the required terminal velocity of the steel ball is $3.27 \times 10^{-5} \text{ ms}^{-1}$

- 17.** Castor oil at 20°C has a coefficient of viscosity 2.42 Nsm^{-2} and density 940 kgm^{-3} . Calculate the terminal velocity of a steel ball of radius 2mm falling under gravity in the oil, taking the density of steel as 7800 kg m^{-3} . [HSEB 2057, 2069, 2072]

Solution:

Given, For Castor oil:

Coefficient of viscosity (η) = 2.42 Nsm^{-2}

Density (σ) = 940 kgm^{-3}

Temperature (T) = 20°C

For steel ball:

Radius (r) = 2 mm = $2 \times 10^{-3} \text{ m}$

Density (ρ) = 7800 kgm^{-3}

Terminal velocity (v) = ?

Since, we know that, when body falls with terminal velocity, then the terminal velocity acquired by the body (from application of stoke's law) is,

Weight of the body = Viscous force + Upthrust

$$\text{or, } mg = 6\pi\eta rv + m'g$$

$$\text{or, } \rho Vg = 6\pi\eta rv + \sigma Vg$$

$$\text{or, } \frac{4}{3} \pi r^3 \rho g = 6\pi\eta rv + \frac{4}{3} \pi r^3 \sigma g$$

$$\text{or, } v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

$$= \frac{2 \times (2 \times 10^{-3})^2 \times (7800 - 940) \times 10}{9 \times 2.42} = 0.025 \text{ ms}^{-1}$$

$$\therefore v = 0.025 \text{ ms}^{-1}$$

Hence, the required terminal velocity of the steel ball is 0.025 ms^{-1} .

- 18.** What is the terminal velocity of a steel ball falling through a tall jar containing glycerin? The densities of the steel ball and glycerin are 8.5 gm/cc and 1.32 gm/cc respectively and the viscosity of the glycerin is 0.85 poise and radius of the steel ball is 2 mm. [HSEB 2059]

Solution:

For glycerin:

Coefficient of viscosity (η) = 0.85 poise

$$0.85 \text{ poise} = \frac{0.85}{10} \text{ Nsm}^{-2} = 0.085 \text{ Nsm}^{-2}$$

[Since, 10 Poise = 1 Nsm^{-2}]

Density (σ) = 1.32 gm/cc

$$= \frac{1.32 \times 10^{-3}}{10^{-6}} \text{ kg m}^{-3} = 1.32 \times 10^3 \text{ kgm}^{-3}$$

For steel ball:

Radius (r) = 2 mm = $2 \times 10^{-3} \text{ m}$

$$\text{Density (}\rho\text{)} = 8.5 \text{ gm/cc} = \frac{8.5 \times 10^{-3}}{10^{-6}} \text{ kg m}^{-3}$$

$$= 8.5 \times 10^3 \text{ kgm}^{-3}$$

Terminal velocity (v) = ?

We know that, when a body falls with terminal velocity, then the terminal velocity acquired by the body (from application of stoke's law) is,

Weight of the body = Viscous force + Upthrust

$$\text{or, } mg = 6\pi\eta rv + m'g$$

$$\text{or, } \rho Vg = 6\pi\eta rv + \sigma Vg$$

$$\text{or, } \frac{4}{3} \pi r^3 \rho g = 6\pi\eta rv + \frac{4}{3} \pi r^3 \sigma g$$

$$\text{or, } v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

$$= \frac{2 \times (2 \times 10^{-3})^2 \times (8.5 \times 10^3 - 1.32 \times 10^3) \times 10}{9 \times 0.085}$$

$$\therefore v = 0.75 \text{ ms}^{-1}$$

Hence, the required terminal velocity of the steel ball is 0.75 ms^{-1} .

19. Calculate the magnitude and direction of the terminal velocity of an 1 mm of radius air bubble using in an oil of viscosity 0.20 Nsm^{-2} and specific gravity of 0.9 and density of air 1.29 kgm^{-3} . [HSEB 2062]

Solution:

For oil:

Coefficient of viscosity (η) = 0.20 Nsm^{-2}

Specific gravity = 0.9

$$\text{Density } (\sigma) = 0.9 \text{ gm/cc} = \frac{0.9 \times 10^{-3}}{10^{-6}} \text{ kgm}^{-3}$$

$$= 0.9 \times 10^3 \text{ kgm}^{-3}$$

[Since, Specific gravity = Density of liquid in cgs]

For an air bubble:

Radius (r) = $1 \text{ mm} = 1 \times 10^{-3} \text{ m}$

Density (ρ) = 1.29 kgm^{-3}

Terminal velocity (v) = ?

We know that, when body falls with terminal velocity, then the terminal velocity acquired by the body (from application of Stoke's law) is,

Weight of body = Viscous force + Upthrust

$$\text{or, } mg = 6\pi\eta rv + m'g$$

$$\text{or, } \rho Vg = 6\pi\eta rv + \sigma Vg$$

$$\text{or, } \frac{4}{3}\pi\rho^3rg = 6\pi\eta rv + \frac{4}{3}\pi r^3\sigma g$$

$$\text{or, } v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

$$= \frac{2 \times (1 \times 10^{-3})^2 \times (1.29 - 0.9 \times 10^3) \times 10}{9 \times 0.020}$$

$$\therefore v = -9.9 \times 10^{-3} \text{ ms}^{-1}$$

The negative sign indicates that, the air bubble moves upward.

Hence, the magnitude of terminal velocity is $9.99 \times 10^{-3} \text{ ms}^{-1}$ and the direction is upward direction.

20. Two drops of same liquid of same radius are falling through air with steady velocity of 2.0 ms^{-1} . If two drops coalesce what would be the terminal velocity? [HSEB 2065]

Solution:

For each drop of same liquid:

Radius (r) = r (say)

Terminal velocity (v) = 2.0 ms^{-1}

For a combined drop (two drop):

Radius (R) = R (say)

Terminal velocity (v) = ?

We know that, when body falls with terminal velocity, then, the terminal velocity acquired by the body (from application of stoke's law) is,

Weight of body = Viscous force + Upthrust

$$\text{or, } mg = 6\pi\eta rv + m'g$$

$$\text{or, } \rho Vg = 6\pi\eta rv + \sigma Vg$$

$$\text{or, } \frac{4}{3}\pi r^3\rho g = 6\pi\eta rv + \frac{4}{3}\pi r^3\sigma g$$

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta} \dots (1). \text{ Where, } \sigma = \text{density of air.}$$

ρ = density of drop of the liquid

For small drop of liquid, we have,

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta} \dots (2)$$

This relation gives the terminal velocity of the small drop. So, for a combined drop of two drops of same liquid, we have,

$$V = \frac{2R^2(\rho - \sigma)g}{9\eta} \dots (3)$$

This relation gives terminal velocity of larger drop.

Dividing equation (3) by (2) we get,

$$\frac{V}{v} = \frac{R^2}{r^2}$$

$$\therefore V = \frac{R^2}{r^2} \times v \dots (4)$$

But, the volume of a combined drop = total volume of two drops.

$$\text{or, } \frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3$$

$$\text{or, } R^3 = 2r^3$$

$$\therefore R = 2^{1/3} \times r \dots (5)$$

From equation (4) and (5) we get,

$$V = \frac{(2^{1/3} \times r)^2}{r^2} \times v = 2^{2/3} \times v = 2^{2/3} \times 2 = 3.17$$

$$\therefore V = 3.17 \text{ ms}^{-1}$$

Hence, the terminal velocity acquired by the combination of two drops of same liquid is 3.17 ms^{-1} .

21. Water flows steadily through a horizontally pipe of non-uniform cross-section. If the pressure of water is $4 \times 10^4 \text{ Nm}^{-2}$ at a point where the velocity of flow is 2 ms^{-1} and cross-section is 0.02 m^2 . What is the pressure at a point where cross-section reduces to 0.01 m^2 and density of air is 1000 kgm^{-3} . [HSEB 2068]

Solution:

Water pressure at the first position (P_1) = $4 \times 10^4 \text{ Nm}^{-2}$

Speed at the first position (v_1) = 2 ms^{-1}

Area at the first position (A_1) = 0.02 m^2

Water pressure at the second position (P_2) = ?

Area at the second position (A_2) = 0.01 m^2

From the equation of continuity,

$$A_1 v_1 = A_2 v_2$$

$$\text{or, } v_2 = \frac{A_1 v_1}{A_2} = \frac{0.02}{0.01} \times 2 = 4 \text{ ms}^{-1}$$

$$\therefore v_2 = 4 \text{ ms}^{-1}$$

From Bernoulli's principle,

$$P_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2$$

[For same horizontal $h_1 = h_2$]

$$\text{or, } P_2 = P_1 + \frac{1}{2} \rho (v_1^2 - v_2^2) = 4 \times 10^4 + \frac{1}{2} \times 1000 \times (2^2 - 4^2)$$

$$\therefore P_2 = 3.4 \times 10^4 \text{ Nm}^{-2}$$

Here, the velocity of flow of water at second position is $3.4 \times 10^4 \text{ Nm}^{-2}$.



Additional Numerical Examples

1. An alloy of mass 588 gm and volume 100 cm^3 is made of iron of density 8.0 gm cm^{-3} and aluminum of density 2.7 gm cm^{-3} . Calculate the proportion
 a. by volume
 b. by mass of the constituents of the alloy.

Solution:

Let, v_i be the volume of iron, then volume of aluminum (v_a) = $100 - v_i$

Mass of iron, (m_i) = $v_i \times 8 = 8v_i \text{ gm}$

Mass of aluminum = $(100 - v_i) \times 2.7 \text{ gm}$
 $= (270 - 2.7 v_i) \text{ gm}$

[Since, sum of masses = $m_i + m_a = 588$]

$$8v_i + 270 - 2.7v_i = 588$$

$$\text{or, } 5.3v_i = 588 - 270$$

$$\text{or, } v_i = \frac{318}{5.3} = 60 \text{ cc}$$

$$\text{or, } v_a = 100 - v_i = 100 - 60 = 40 \text{ cc}$$

$$\text{a. } v_i : v_a = 60 : 40 \text{ cc}$$

Hence, the proportion by volume is $\frac{3}{2}$ or, 3 : 2.

$$\text{b. Mass of iron } (m_i) = 8v_i = 8 \times 60 = 480 \text{ gm.}$$

$$\text{Mass of aluminum } (m_a) = 40 \times 2.7 = 108 \text{ gm}$$

$$\therefore m_i : m_a = 480 : 108 = 40 : 9$$

Hence, the proportion by mass is 40:9.

2. A 25 cm thick block of ice floating on fresh water can support an 80 kg man standing on it, what is the smallest area of the ice block? [Density of ice = 0.917 g/cm^3]

Solution:

Thickness of a block of ice = 25 cm

If A is the area of the block of ice, its volume is given by $V = 25A$

mass of the ice block = $25A \times 0.917$

Weight of man and block of ice

$$= (80 \times 1000 + 25A \times 0.917) \text{ g} \dots (1)$$

$$\text{Weight of water displaced} = (25A \times 1) \text{ g} \dots (2)$$

$$\text{From law of floatation, } (80,000 + 25A \times 0.917) \text{ g} = 25A \times \text{g}$$

$$A = 40,000 \text{ cm}^2 = 4 \text{ m}^2$$

Hence, the smallest area of the ice block is 4 m^2

3. A spherical liquid drop of diameter $2 \times 10^{-4} \text{ m}$ is falling freely through air. Find the terminal velocity of the drop. The density of liquid is 500 kg/m^3 and the viscosity of air is $2 \times 10^{-5} \text{ N s/m}^2$. Also find the viscous force acting on the drop. [Neglect air density]

Solution:

$$\text{According to stoke's law } 6\pi\eta r v = \frac{4}{3} \pi r^3 \rho g$$

$$\text{Terminal velocity } v = \frac{2}{9} \cdot \frac{r^2 \rho g}{\eta}$$

$$\text{Here, } r = \frac{2 \times 10^{-4}}{2} \text{ m} = 10^{-4} \text{ m}$$

$$\rho = 500 \text{ kg/m}^3, g = 9.8 \text{ m/s}^2, \eta = 2 \times 10^{-5} \text{ N s/m}^2$$

$$\text{So, } v = \frac{2}{9} \times \frac{(1 \times 10^{-4})^2 \times 500 \times 9.8}{2 \times 10^{-5}} = 0.54 \text{ m/s}$$

Viscous force acting on the drop

$$F = 6\pi\eta r v = 6 \times \frac{22}{7} \times 2 \times 10^{-5} \times 10^{-4} \times 0.54$$

$$= 2.04 \times 10^{-8} \text{ N}$$

4. A square plate of 0.1 m side moves parallel to another plate with velocity of 0.1 m/s. Both plates immersed in water. If the viscous force is 0.002 N and viscosity of water is 0.1 poise, what is their distant apart?

Solution:

$$\text{Area of plate } A = 0.1 \times 0.1 = 0.01 \text{ m}^2$$

$$\text{Relative velocity } dv = 0.1 \text{ m/s}$$

$$\text{Coefficient of viscosity } \eta = 0.1 \text{ poise} = 0.01 \text{ decapoise}$$

$$\text{Viscous force } F = 0.002 \text{ N}$$

$$\text{Now, } F = \eta A \frac{dv}{dx}$$

$$\text{or, } 0.002 = 0.01 \times 0.01 \times \frac{0.1}{dx}$$

$$\text{or, } dx = \frac{1 \times 10^{-5}}{0.002} = 5 \times 10^{-3} \text{ m}$$

5. Two identical drops of water are falling through air with a steady velocity of 10 cm s^{-1} . If the drops combine to form a single drop, what would be the terminal velocity of the single drop?

Solution:

$$\text{Terminal velocity} \propto (\text{radius})^2$$

Let r is the radius of each of small drop and R is the

radius of bigger drop.

Let V_T and $V_{T'}$ be the terminal velocities of the drops before and after combination respectively.

$$\text{Then, } \frac{V_T}{V_{T'}} = \frac{r^2}{R^2} \quad [\text{From question}]$$

$$\text{or, } 2 \times \left(\frac{4}{3} \pi r^3 \right) = \frac{4}{3} \pi R^3$$

$$\text{or, } \left(\frac{r}{R} \right)^2 = \frac{1}{2^{2/3}}$$

$$\begin{aligned} \text{Hence, } V_{T'} &= V_T \left(\frac{R^2}{r^2} \right) = 10 \times 2^{2/3} \\ &= 10 \times 1.59 = 15.9 \text{ cm/s} \end{aligned}$$

6. If the velocity of water in a 6 m diameter pipe is 5 m/s, what is the velocity in a 3 m diameter pipe which connects with it, both pipes flowing full?

Solution:

$$\text{Given: } r_1 = \frac{6}{2} = 3 \text{ m} \quad r_2 = \frac{3}{2} = 1.5 \text{ m}$$

$$v_1 = 5 \text{ m/s} \quad v_2 = ?$$

According to equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$\pi r_1^2 v_1 = \pi r_2^2 v_2$$

$$v_2 = \frac{r_1^2 v_1}{r_2^2} = \frac{(3)^2 \times 5}{(1.5)^2} = 20 \text{ m/s}$$

7. Water flows along a horizontal pipe of cross-section area 50 cm^2 . The speed of the water is 14 ms^{-1} but this rises to 75 ms^{-1} in a constriction in the pipe. what is the area of this narrow part of the tube?

Solution:

$$\text{Here, } A_1 = 50 \text{ cm}^2 \quad v_1 = 14 \text{ ms}^{-1},$$

$$v_2 = 75 \text{ ms}^{-1} \text{ and } A_2 = ?$$

Now from the equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$\text{or, } A_2 = \frac{A_1 v_1}{v_2} = \frac{50 \times 14}{75} = 9.33 \text{ cm}^2$$

8. Water flows through a horizontal pipe having a tapering bore. The velocity of water is 2 m/sec at the broader end and the pressure is $1 \text{ kilo-Newton/m}^2$ less at the narrow end. What is the velocity of water at the latter end?

Solution:

Let v be the velocity of water at the latter end.

$$\text{Here, } v_1 = 2 \text{ m/s } v_2 = v \text{ m/s } p_1 - p_2 = 1000 \text{ N/m}^2$$

From Bernoulli's principle

$$\frac{p_1}{\rho} + gh + \frac{1}{2} v_1^2 = \frac{p_2}{\rho} + gh + \frac{1}{2} v_2^2 \quad [\text{for horizontal pipe}]$$

$$\text{or, } \frac{1}{2} (v_2^2 - v_1^2) = \frac{p_1 - p_2}{\rho}$$

$$\text{or, } \frac{1}{2} (v^2 - 2^2) = \frac{1000}{1000}$$

$$\text{or, } v^2 = 2 + 4 = 6$$

$$\therefore v = \sqrt{6} = 2.45 \text{ m/s}$$

9. What is the maximum weight of an aircraft with a wing area of 50 m^2 flying horizontally, if the velocity of the air over the upper surface of the wing is 150 ms^{-1} and that over the lower surface is 140 ms^{-1} ?

[Density of air = 1.29 kgm^{-3}]

Solution:

Neglecting the gravitational potential difference between the top and the bottom of the wing, Bernoulli's equation becomes

$$\frac{p_1}{\rho} + \frac{v_1^2}{2} = \frac{p_2}{\rho} + \frac{v_2^2}{2}$$

$$\text{or, } p_2 - p_1 = \frac{\rho}{2} (v_1^2 - v_2^2)$$

$$\text{Here, } v_1 = 150 \text{ m/s and } v_2 = 140 \text{ m/s}$$

$$\therefore p_2 - p_1 = \frac{\rho}{2} (v_1^2 - v_2^2) = \frac{1.29}{2} (150^2 - 140^2) = 1870.5 \text{ N/m}^2$$

$$\begin{aligned} \text{Maximum lifting force} &= \text{Pressure difference} \times \text{area} \\ &= 1870.5 \times 50 = 93525 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Maximum weight of the aircraft (in kg)} &= \frac{93525}{9.8} \\ &= 9543.36 \text{ kg} \end{aligned}$$

10. Water at a pressure of 3.3 atm at street level flows into an office building at a speed of 0.5 m/s through a pipe 5 cm diameter. The pipes taper down to 2.5 cm diameter by the top floor, 25 m above. Calculate the flow velocity and the pressure in such a pipe on the top floor.

Solution:

$$\text{Here } P_1 = 3.3 \text{ atm} \quad p_2 = ?$$

$$v_1 = 0.5 \text{ m/s} \quad v_2 = ?$$

$$d_1 = 5 \text{ cm} \quad d_2 = 2.5 \text{ cm}$$

$$\text{and, } h_2 - h_1 = 25 \text{ m}$$

Now from the equation of continuity,

$$A_1 v_1 = A_2 v_2$$

$$\text{or, } v_2 = \frac{A_1}{A_2} v_1 = \frac{\frac{\pi}{4} d_1^2}{\frac{\pi}{4} d_2^2} \times v_1$$

$$= \left(\frac{d_1}{d_2} \right)^2 \times v_1 = \left(\frac{5}{2.5} \right)^2 \times 0.5 = 2 \text{ m/s}$$

From Bernoulli's equation

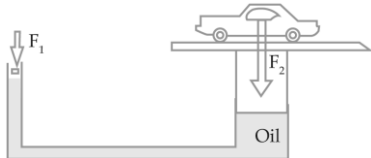
$$P_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2$$

$$\begin{aligned} \text{or, } P_2 &= P_1 + \rho g (h_1 - h_2) + \frac{1}{2} \rho (v_1^2 - v_2^2) \\ &= 3.3 \times 10^5 + 1000 \times 9.8 \times (-25) + \frac{1}{2} \times 1000 (0.5^2 - 2^2) \\ &= 83125 \text{ N/m}^2 = 0.83 \times 10^5 \text{ N/m}^2 = 0.83 \text{ atm} \end{aligned}$$

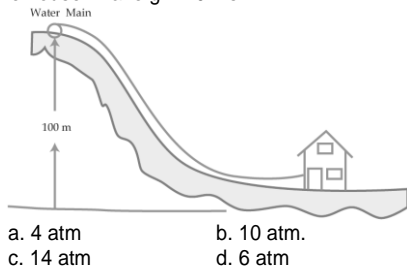
Exercise

A. Multiple Choice Questions

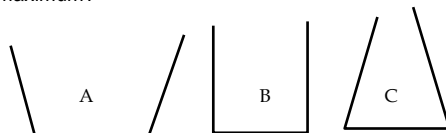
Circle the best alternative to the following questions:

- A body is just floating in a liquid. If the body is pressed down and released then it will
 - sink
 - come back to initial position
 - oscillate with S.H.M
 - remain in new position
- For stable equilibrium of a floating body, its centre of buoyancy should be
 - vertically above its centre of gravity
 - horizontally in line with its centre of gravity
 - below its centre of gravity
 - anywhere
- The force of buoyancy depends upon
 - shape of body
 - mass of body
 - the mass of liquid displaced
 - the depth to which body is immersed
- A body weighs 60 kg in air and 40 kg in water. The specific gravity of body is
 - 3
 - 1.5
 - 2×10^3
 - 0.5
- How much lead of specific gravity 11 should be added to piece of cork of specific gravity 0.2 weighing 10 g so that it may just float on water?
 - 44 g
 - 4.4 g
 - 440 g
 - 2.2 g
- For a body floating in water, the apparent weight is equal to
 - actual weight
 - weight of liquid displaced
 - zero
 - weight of liquid displaced
- What fraction of wooden raft of density of 0.8 g/cc will be outside the sea water having density 1.2 g/cc immersed?
 - $\frac{2}{3}$
 - $\frac{1}{3}$
 - $\frac{2}{5}$
 - $\frac{1}{5}$
- An automobile rack is lifted by a hydraulic jack that consists of two pistons connected by a pipe as shown in figure. The large piston is 1 m in diameter and the small piston is 10 cm in diameter. If the weight of the car is F_2 , how much smaller a force is required on the small piston to lift the car?
 
 - F_2
 - $0.1F_2$
 - $0.01F_2$
 - $0.001F_2$
- One thousand small water drops of equal size combine to form a big drop. The ratio of final surface energy to total initial surface energy is
 - 10:1
 - 1:10
 - 1000:1
 - 1:100
- If a liquid does not wet the solid surface, the angle of contact is
 - acute
 - right angle
 - obtuse
 - none
- Work done in blowing a soap bubble of radius R is w. The work done in increasing its radius from R to 3R will be
 - 2w
 - 4w
 - 8w
 - 9w
- A disc of paper of radius 'R' has a hole of radius r. It is floating on liquid of surface tension T, the force of surface tension on the disc is
 - $T \cdot 2\pi(R + r)$
 - $T \cdot 4\pi(R + r)$
 - $T \cdot 2\pi R$
 - $T \cdot 2\pi(R - r)$
- The height up to which water rise in a capillary will be.
 - maximum when water temperature is 4°C
 - minimum when water temperature is 4°C
 - minimum when water temperature is 0°C
 - same at all temperature
- Two soap bubbles have radii in the ratio 5:1. The ratio of excess pressure inside these bubbles is
 - 1:5
 - 5:1
 - 1:25
 - 25:1
- A soap bubble of radius 'r' is formed with in soap solution. The excess pressure inside the bubble is
 - $\frac{1T}{r}$
 - $\frac{2T}{r}$
 - $p + \frac{2T}{r}$
 - $p - \frac{4T}{r}$
- Surface tension arises due to
 - adhesive force between the molecules
 - cohesive force between the molecules
 - all of the above
 - none of above
- When salt is added to pure water, surface tension
 - increases
 - decreases
 - remains unchanged
 - becomes zero
- When detergent is added to pure water, surface tension
 - increases
 - deceases
 - remains unchanged
 - becomes zero
- Water rises in a capillary to be to a height H. When the capillary is vertical, the vertical height of water level in it will
 - Increase
 - decrease
 - will not change
 - rise to entire length of tube
- The rise of liquid in a capillary tube doesn't depend upon
 - density of liquid
 - atmospheric pressure
 - angle of contact
 - radius of tube

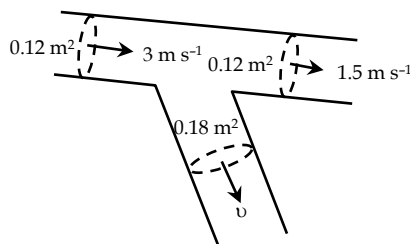
21. Bernoulli's theorem is important in the field of
 a. electric circuit b. magnetism
 c. flow of fluid d. photoelectric effect
22. Bernoulli's theorem is based on
 a. conservation of momentum
 b. conservation of energy
 c. conservation of mass
 d. mass - energy equivalence
23. In a wide river, the velocity of water at the middle
 a. increases with depth b. is the same everywhere
 c. decreases with depth d. is zero
24. Eight drops of equal size are falling through air with steady velocity of 10 m/s. If the drop coalesces, what would be the terminal velocity of bigger drop?
 a. 10 cm/s b. 20 cm/s
 c. ∞ d. 0
25. Rain drop falls with constant velocity due to
 a. viscosity b. buoyancy
 c. gravity d. surface tension
26. A rain drop of radius 'r' has terminal velocity v m/s in air. Viscous force on it is F . If the radius of drop is $2r$ and the drop falls with terminal velocity in same air, the viscous force on it will be
 a. F b. $F/2$
 c. $4F$ d. $8F$
27. A house is connected to a city water main that is 100 m above the house in altitude as shown in figure. If the city water pressure is 4 atm, what will be the water pressure at the house? Take $g = 10 \text{ ms}^{-2}$.



28. The three vessels shown in figure below have same base area. Equal volumes of a liquid are poured in the three vessels. In which vessel, the pressure on the base will be maximum?



- a. A b. B
 c. C d. All have equal pressure
29. An incompressible liquid travels as shown in figure below. Calculate the speed of the liquid in the lower branch.



- a. 1 ms^{-1} b. 1.5 ms^{-1}
 c. 2 ms^{-1} d. 3 ms^{-1}
30. A liquid is contained in a vertical tube of semi-circular cross-section as shown in figure. What is the ratio of the force of surface tension on the curved part and the flat part of the tube? The angle of contact is zero.



- a. π b. $\pi/2$
 c. 2π d. 1

Answers Key

1. a	2. a	3. c	4. a	5. a	6. c	7. d	8. c	9. b	10. c
11. c	12. a	13. b	14. a	15. b	16. b	17. a	18. b	19. c	20. b
21. c	22. b	23. c	24. c	25. a	26. d	27. c	28. c	29. a	30. b

B. Short Questions

- State the laws of floatation.
- Does a ship sink more in river water or in sea water?
- Why is the bottom of a ship made heavy?
- Why does ice float in water?
- Why is it easier to lift a body in a liquid than in air?
- What will happen to the level of water in a container when a submerged piece of ice melts completely?
- Define angle of contact.
- Define the term capillary.
- Why mercury does not wet the glass tube?
- What is the principle on which Bernoulli's theorem based?
- What is the principle on which continuity equation based?
- What do you mean by viscosity?

13. State Stoke's law for viscosity.
14. How do the leaves of tree get water from ground?
15. Lead has a greater density than iron and both are denser than water. Is the buoyant force on a lead object greater than, less than or equal to the buoyant force on an iron object of the same volume?
16. Steel balls sink in water but they don't sink in mercury. Why?
17. How will you make difference between density and specific gravity of a body?
18. Why do bigger air bubbles rise faster than the smaller ones in boiling water?
19. The purity of the gold can be tested by weighting it in air and water. How?
20. In still air, a helium filled balloon rises upto a certain height and then stops rising. Why?
21. An ice cube floats in a glass of water. When the ice melts, will the water level in the glass rise, fall or remain unchanged? Explain.
22. Explain why does water rise in a capillary, while mercury does not?
23. Soap helps in cleaning the clothes. Explain.
24. Why hot soup is more tasteful than cold one?
25. Why insect are killed by spraying kerosene oil on the surface of water in ponds?
26. Why is it very difficult to separate two plates containing a thin layer of water between them?
27. If the length of capillary is less than that required, will the water fountain come out of the capillary? Explain.
28. Why are falling rains spherical?
29. Why does a small quantity of liquid assume spherical shape?
30. Explain why the surface of water in a glass tube is concave while that of mercury is convex.
31. Define angle of contact, explain why the angle of contact for Hg is obtuse and that for water is acute.
32. What are cohesive and adhesive forces?
33. We use towels to dry our body after taking a shower, why?
34. Hairs of a brush spread out when it is dipped in water and cling together as soon as it is taken out of water. Explain.
35. When a fluid flows through a narrow constriction, its speed increases. How does it get the energy for this extra speed?
36. If two ships moving parallel and close to each other they experience an attractive force. Why?
37. Why does a flag flutter when strong winds are blowing?
38. During a certain wind storm light roof of home etc. are blown off. Why?
39. Why do the clouds float in air?
40. Why the pressure decreases when water flowing into a broader pipe enters a narrower pipe?
41. What is the difference between stream line and turbulent motion in liquids?
42. Airports at high elevations have longer runways for take offs and landing than do airports at sea level, why?
43. Express the dimension of the velocity gradient from the definition of the coefficient of viscosity.
44. Machine parts are jammed in cold day, why?
45. Explain with diagram, the meaning of velocity gradient in the case of liquid flowing in a tube.

C. Long Questions

1. State Archimedes principle and deduce it from the laws of liquid pressure.
2. State the law of floatation and show how it follows from Archimedes principle. State the conditions of equilibrium and of the stability of a floating body.
3. Explain the term centre of buoyancy and meta centre. Why should the meta centre lie above the centre of

gravity of a floating body?

4. Define specific gravity. How can you relate specific gravity and density of any substance?
5. Explain surface tension. Discuss molecular theory of surface tension.
6. Prove that the surface energy and surface tension are numerically same. Explain the concept of the angle of contact, with necessary figure, when the surface of the liquid is convex if viewed from the above.
7. What is viscosity? Obtain an expression of viscous force from Newton's laws for viscosity. [HSEB 2066]
8. Using dimensional consideration, deduce poiseuille's formula for the rate of flow of a liquid through a capillary tube. [HSEB 2058]
9. How is the coefficient of viscosity related with the velocity gradient of the flowing liquid? Use dimensional method to obtain poiseuille's formula for the flow of fluid. [HSEB 2065]
10. State stoke's law and deduce it from dimensional analysis. Define coefficient of viscosity of a liquid.
11. What is terminal velocity? Using stoke's formula find the expression for the terminal velocity. What is the acceleration of the body inside the liquid before it attains terminal velocity. [HSEB 2069]
12. Derive an expression for terminal velocity of a small spherical ball dropped gently in a viscous liquid.
13. State and prove Bernoulli's theorem for the steady flow of an incompressible and non viscous flow. [HSEB 2059, 2063, 2064, 2070, 2070S, 2072]
14. Describe stoke's method to find the coefficient of viscosity of a liquid in the laboratory with necessary theory. [NEB 2074]

D. Numerical Questions

1. The weight of a body in water is one third of its weight in air. What is the density of the material of the body? [Ans: $1.5 \times 10^3 \text{ kg m}^{-3}$]
2. A piece of pure gold of density 19.3 gm cm^{-3} is suspected to be hollow from inside. It weighs 38.250 gm in air and portion of the gold, if any? [Ans: 2.403 cm^3]
3. A rectangular tank 10 m long, 3 m breadth and 2 m deep is full of water. Find the thrust on each face. [Ans: $5 \times 10^4 \text{ m}$]
4. A piece of wood of relative density 0.81 . What is the percentage of volume of the wood above the surface of the oil? [Ans: 69.2%]
5. A 1 kg body made of the material of density 8000 kg m^{-3} is suspended by a string so that the body is totally immersed in a liquid of density 800 kg m^{-3} . What is the tension in the string? [Ans: 9 N]
6. A soap film is formed on a rectangular frame of 7 cm . dipping into a soap solution. The frame hangs from the arm of a balance. An extra weight of 0.4 gm . is to be placed in the opposite pan to balance the pull on the frame. Calculate the surface tension of the soap solution. [Ans: 0.028 Nm^{-1}]
7. A glass plate has length 10 cm , breadth 1.5 cm , and thickness 0.2 cm , weight 8.2 gm . in air. It is held vertically with long side horizontal and half the plate immersed in water. What will be its apparent weight? Surface tension of water = 75 dynes/cm . [Ans: 8.219 gm wt.]
8. Calculate the work done in breaking a drop of water of 2 mm . diameter into one thousand million droplets all of the same size. Surface tension of water is $72 \times 10^{-3} \text{ Nm}^{-1}$. [Ans: $9.042 \times 10^{-4} \text{ J}$]
9. What amount of energy will be liberated if 1000 droplets of water each of diameter 10^{-6} cm . coalesce to form a bigger drop. Surface tension of water is $75 \times 10^{-3} \text{ Nm}^{-1}$. [Ans: $2.12 \times 10^{-14} \text{ J}$]
10. A capillary tube of 1 mm diameter and 20 cm long is fitted horizontally to a vessel kept full of alcohol of density 0.8 gm/ c.c. The depth of centre of capillary tube below the surface of alcohol is 20 cm . If the viscosity of oil is 0.12 Poise , find the amount of liquid that will flow in 5 minutes . [Ans: 3.85 gms.]
11. A plate of metal 100 cm^2 in area rests on a layer of castor oil 2 mm thick whose coefficient of viscosity is 15.5 poise. Calculate the horizontal force required to move the plate with a uniform speed of 3 cm s^{-1} . [Ans: 23250 dynes]

12. A metal plate 5 cm square rests on a 1 mm thick castor oil layer. If a force of 22500 dyne is needed to move the plate with a velocity 3 cm s^{-1} , calculate the coefficient of viscosity of the castor oil. [Ans. 15 poise]
13. Water is escaping from a cistern through a horizontal narrow tube 11 cm long and 0.2 mm in radius. If the level of water in the cistern always remains at a height of 42 cm above the centre of tube, calculate the volume of liquid flowing out of the tube in 2 minutes. Given coefficient of viscosity of water is 0.01 poise. [Ans. 2.82 cm^3]
14. A capillary tube of diameter 1 mm and length 15 cm is fitted horizontally to a vessel kept full of alcohol of density 0.8 g cm^{-3} . The depth of the centre of the tube is 25 cm. If the viscosity of alcohol is 0.12 ergs units, find the amount of liquid that will flow in 5 minutes. [Ans. 64.16 g]
15. Water flows through a tube of 2 mm diameter and 50 cm length under a pressure of $10^5 \text{ dyne cm}^{-2}$. If the viscosity of water is 1.0 centi poise, calculate the rate of flow (volume per second) and the speed of the water coming out of the tube. [Ans. $0.786 \text{ cm}^3 \text{ s}^{-1}$, 25 cm s^{-1}]
16. Determine the radius of a drop of water falling through air, if it covers 4.1 cm in 4 second with a uniform velocity. Given η for air is 1.8×10^{-4} poise. [Neglect the density of air]. [Ans. $9.96 \times 10^{-3} \text{ cm}$]
17. An air bubble of 1 cm radius is rising at a steady rate of 5 mm s^{-1} through liquid of density 0.8 g cm^{-3} . Calculate the coefficient of viscosity of the liquid. [Neglect the density of air]. [Ans. 348.4 poise]
18. Two equal drops of water are falling through air with a steady velocity 10 cm s^{-1} . If drops recombine to form a single drop, what will be new terminal velocity? [Ans. 15.88 cm s^{-1}]
19. Emery powder particles are stirred up in a beaker of water 0.1 m deep. Assuming the particles to be spherical and of oil sizes, calculate the radius of the largest particle remaining in suspension after 24 hours. Given that density of emery is 4000 kg m^{-3} and coefficient of viscosity of water is 0.001 decapoise. [Ans. $4.208 \times 10^{-7} \text{ m}$]
20. A tank containing water has an orifice in one vertical side. If the center of the orifice is 4.9 meters below the surface level in the tank, find the velocity of discharge, assuming that there is no wastage of energy. [Ans. 9.8 ms^{-1}]
21. A pipe is running full of water. At a certain point A, it tapers from 60 cm to 20 cm diameter at B. The pressure difference at A and B is 100 cm of water column. Find the rate of flow of water through the pipe. [Ans. $1.4 \times 10^5 \text{ c.c./sec.}$]
22. Water flows through a horizontal pipe of varying cross - section at the rate of 30 litres per minute. Determine the velocity of water at the point where diameter is 6 cm. [Ans. 0.1769 ms^{-1}]
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